Quantum phases and an anomaly of interacting fermionic atoms

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Two topics:

I. Breached pair superfluidity. [with F. Wilczek, P. Zoller, E. Gubankova, M. Forbes]

II. A quantum anomaly---chiral mass flow of atoms in p-wave resonance. [myself]

Breached pair superfluidity (BP)

Collaborators:

M. Forbes (MIT graduate)
E. Gubankova (MIT postdoc)
F. Wilczek (MIT)
P. Zoller (Innsbruck)

publications

1. PRL **90**, 047002 (2003)
2. PRL **91**, 032001(2003)
3. PRA **70,** 033603 (2004)
4. PRL **94**, 017001 (2005)

News story: "Odd particle out",
Phys. Rev. Focus (January 5, 2005; story 1)

Motivation: atomic Fermi gases

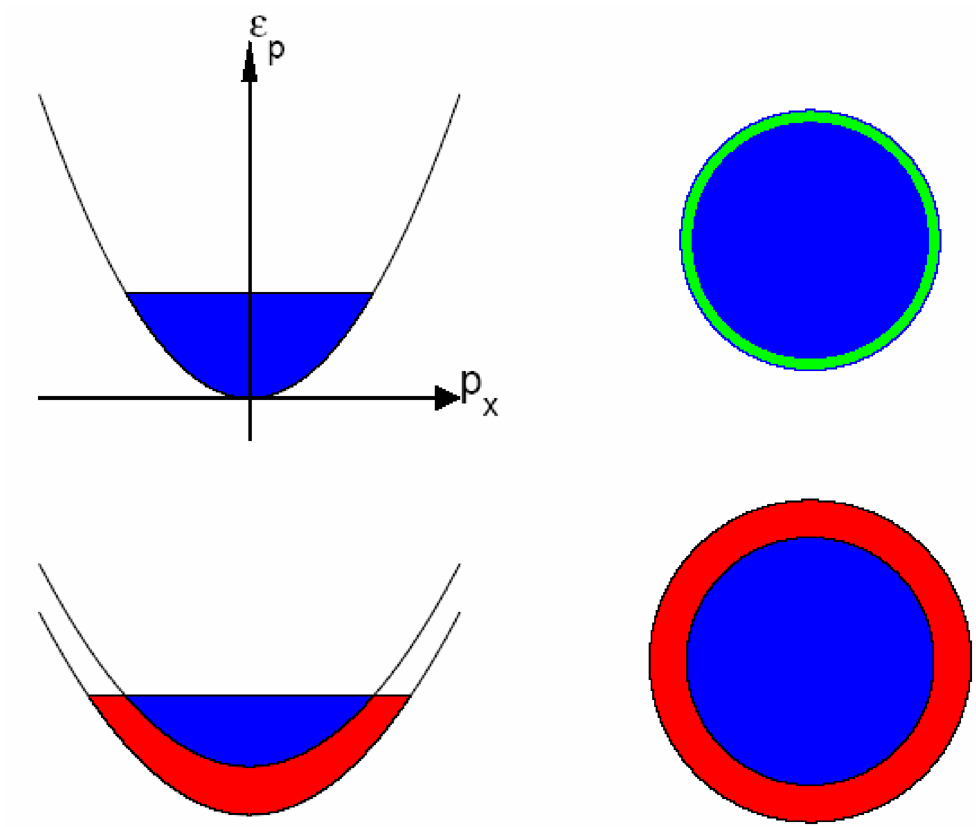
- BCS superfluidity of fermionic atoms
- BEC of molecules, BEC/BCS crossover, resonace models
- Pairing with mismatched fermi surfaces:
  - Two spin components are separately conserved; different densities
  - new pairing possibility? — “breached pairing”
  - on-earth “atomic” simulator for color superconductivity in nuclear matter? (mismatched fermi surface in quark matter in neutron stars)

Different kinds of pairing

BCS	Bardeen-Cooper-Schrieffer (1957)
LOFF	Larkin and Ovchinnikov; independently Fulde and Ferrell (1964)
Breached pairing	Pairing occurs within the interior or exterior of a large Fermi ball. [This talk!].

Heuristic introduction to BP

Recall BCS pairing



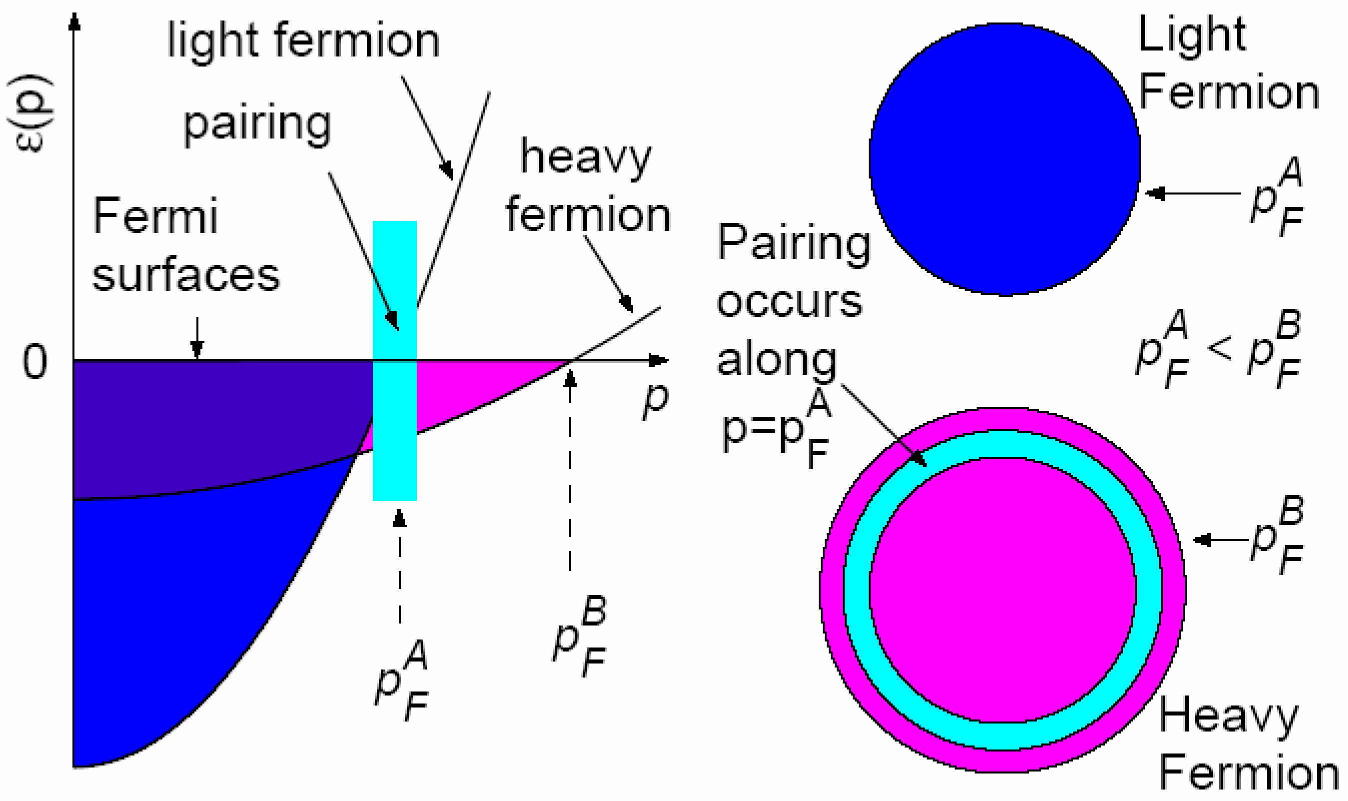
momentum gap

$$\kappa = \frac{\Delta}{v_F}$$

$$\delta p_F \equiv p_F^{\downarrow} - p_F^{\uparrow}$$

when $\delta p_F \gtrsim \kappa$,
BCS impossible!

Breached Pair Superfluidity (BP)



[WVL, F. Wilczek, PRL (2003)]

BP state = a superfluid + a normal Fermi liquid at T=0; has gapped and gapless quasiparticle excitations.

Mean field theory of BP

Model:

$$H = \sum_{\mathbf{p}} \epsilon_{\mathbf{p}\alpha} \psi^\dagger_{\mathbf{p}\alpha} \psi_{\mathbf{p}\alpha} + \sum_{\mathbf{p}\mathbf{p}'} V(\mathbf{p}-\mathbf{p}') \psi^\dagger_{\mathbf{p}\uparrow} \psi^\dagger_{-\mathbf{p}\downarrow} \psi_{-\mathbf{p}'\downarrow} \psi_{\mathbf{p}'\uparrow}$$

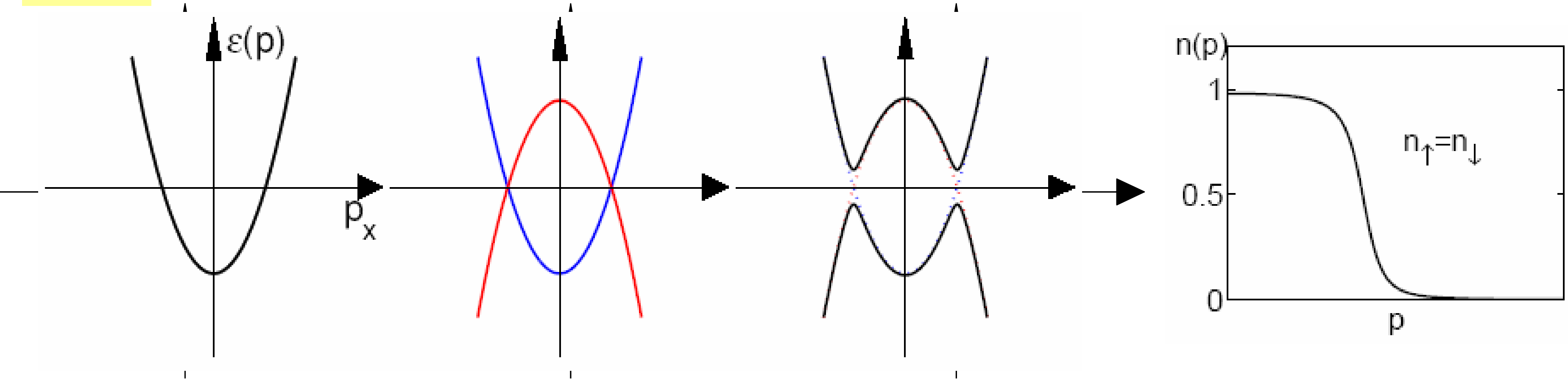
$$\epsilon_{\mathbf{p}\alpha} = \frac{\mathbf{p}^2}{2m_\alpha} - \mu_\alpha \,, \qquad \alpha = \uparrow, \downarrow$$

For real-space
δ-like interaction

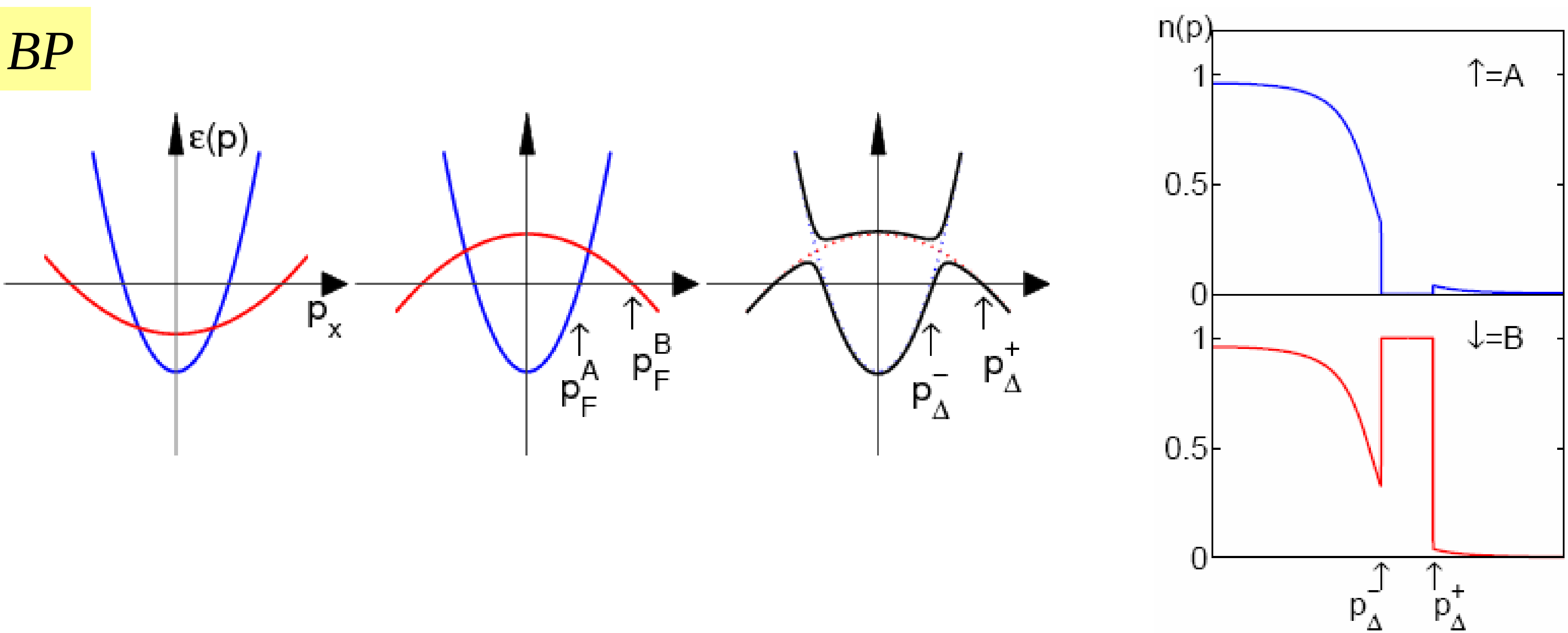
$$\Longrightarrow \quad V(\mathbf{q}) = g = \frac{4\pi\hbar^2 a_s}{m} = const$$

$$(a_s < 0)$$

BCS


ε(p)
p$_x$
n(p)
1
0.5
0
n$_\uparrow$=n$_\downarrow$
p

BP


ε(p)
p$_x$
p$_F^A$
p$_F^B$
p$_\Delta^-$
p$_\Delta^+$
n(p)
1
0.5
0
↑=A
↓=B
p$_\Delta^-$
p$_\Delta^+$

Many body wavefunction

BCS vs BP

Unpaired matter??

$$
\begin{aligned}
|BCS\rangle &= \prod_{\mathbf{p}} (u_{\mathbf{p}} + v_{\mathbf{p}} \psi^{\dagger}_{\mathbf{p}\uparrow} \psi^{\dagger}_{-\mathbf{p}\downarrow}) |0\rangle \\
|BP\rangle &= \prod_{\mathbf{p}\notin \mathrm{breach}} (u_{\mathbf{p}} + v_{\mathbf{p}} \psi^{\dagger}_{\mathbf{p}\uparrow} \psi^{\dagger}_{-\mathbf{p}\downarrow}) \boxed{\prod_{\mathbf{p}\in \mathrm{breach}} \psi^{\dagger}_{\mathbf{p}\downarrow}} |0\rangle
\end{aligned}
$$

where
$$
\begin{pmatrix} u^2_{\mathbf{p}} \\ v^2_{\mathbf{p}} \end{pmatrix} = \frac{1}{2} \left(1 \pm \frac{\epsilon^{+}_{\mathbf{p}}}{\sqrt{{\epsilon^{+}_{\mathbf{p}}}^2 + \Delta^2_{\mathbf{p}}}} \right)
$$

"breach" region:
$$
p^{-}_{\Delta} \leq |\mathbf{p}| \leq p^{+}_{\Delta}
$$

How stable?

The stability of BP criticized by:

1. Shin-Tza Wu, Sungkit Yip, PRA (2003)
2. P. F. Bedaque, H. Caldas, G. Rupak, PRL (2003); Caldas, PRA (2004)

Both are correct, but are done for a short-range delta-interaction.

Stability issue overcome and clarified in:

our latest [PRL 94, 017001 (2005)]

Need

1. a finite or long range interaction; or

2. a momentum cutoff

$$\underset{\text{effective range}}{R^*} \gtrsim \underset{\text{inter-atom distance}}{k_F^{-1}}$$

Effective range in real atomic gases

From D. Petrov, talk given at KITP Conference: Quantum gases 2004:

	R_e [Å]	B_0 [G]	Δ_B [G]	$\partial E_{res}/\partial B$	a_{bg} [Å]	R^* [Å]
^{6}Li	30	543.25	0.1	$2\mu_B$	32	19000
^{23}Na	45	907	1	$3.7\mu_B$	33	260
^{87}Rb	85	1007.4	0.17	$2.5\mu_B$	60	320
^{133}Cs	100	19.8	0.005	$0.55\mu_B$	160	13000

[http://online.itp.ucsb.edu/online/gases_c04/petrov/]

Gas density: $n \sim 10^{14}\mathrm{cm}^{-3} \quad \Rightarrow \quad k_F^{-1} \sim 1.0\mu\mathrm{m}$

Case of strong coupling, short-range interaction, and equal mass

Quantum Monte Carlo results found:

A homogeneous, spin-polarized gapless superfluid [that is a BP] is favored against phase separation in real space.

[J. Carlson, Sanjay Reddy, **cond-mat/0503256]**

How to realize in atomic gases

A. Hetero-nuclear mixture of two species

$$^6\mathrm{Li} + {}^{40}\mathrm{K}\,,\ {}^6\mathrm{Li} + {}^{86}\mathrm{Rb}\,,\ \ldots$$

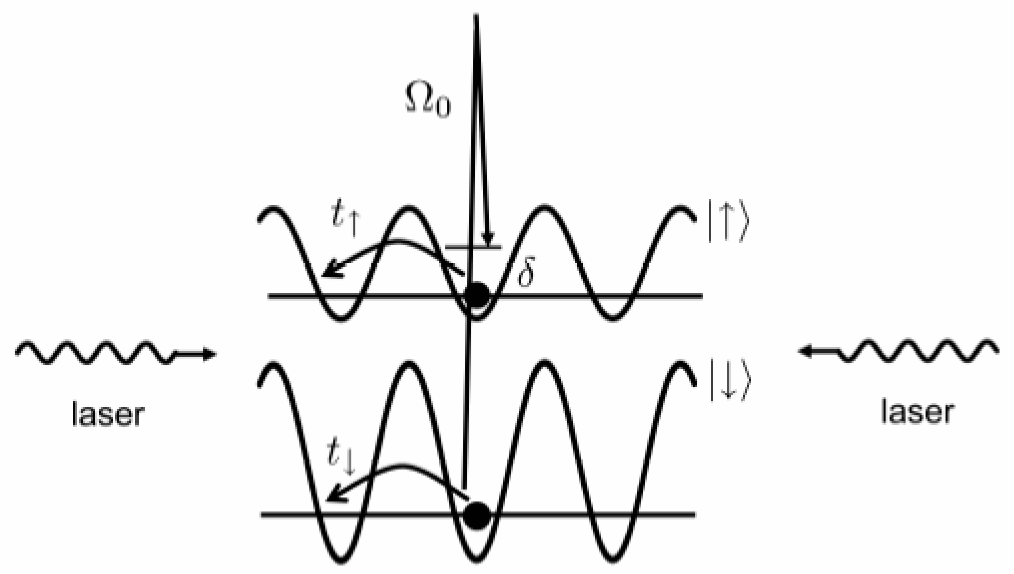
Make two species of unequal densities!

Hetero-nuclear resonance to generate attractive interactions.

B. Lattice atomic gases

Proposed experiment of fermionic atoms on lattice

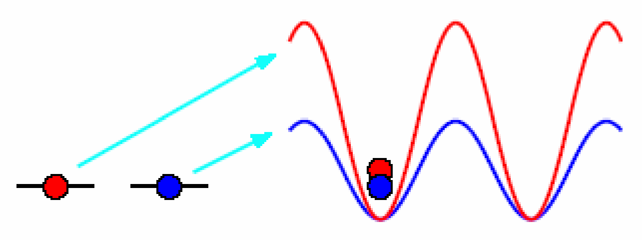
[WVL, F. Wilczek, and P. Zoller, PRA (2004)]



incoherent & different densities or coherent by Rabbi oscillation but detuned

⇓

mismatched fermi surfaces



hopping matrix elements:

$$t_\uparrow \gg t_\downarrow\,, \quad t_\alpha \propto \frac{1}{m_\alpha}$$

Key features of Breached Pair

- coexisting superfluid and normal components at T=0;
- phase separated in momentum space;
- both gapped and gapless quasiparticle excitations.

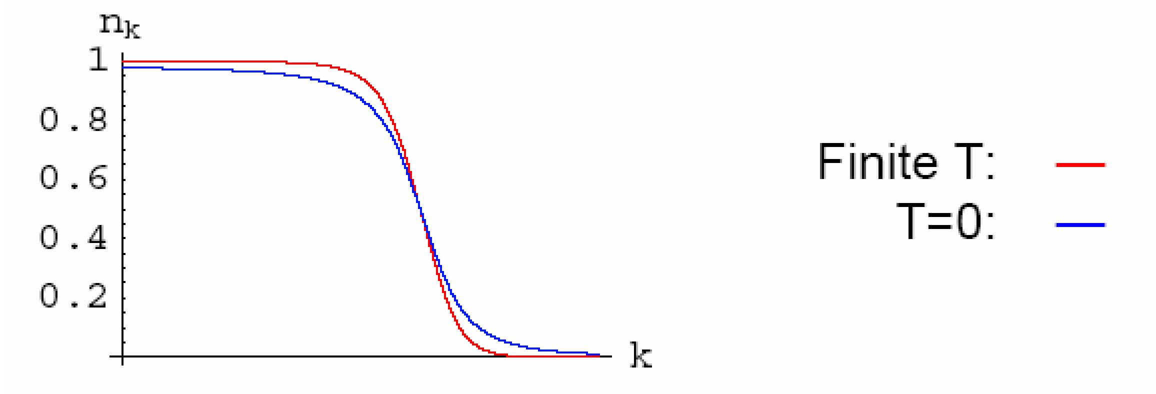
relevances to reality

- realizable with cold atoms;
- may occur as a color superconductor in quark matter such as neutron stars
- "… other scenarios for uncondensed electrons should be considered, such as 'interior gap [BP] superfluidity'" for the heavy-fermion superconductor CeCoIn5 [*quote* M. A. Tanatar, Louis Taillefer, et al. cond-mat/0503342]

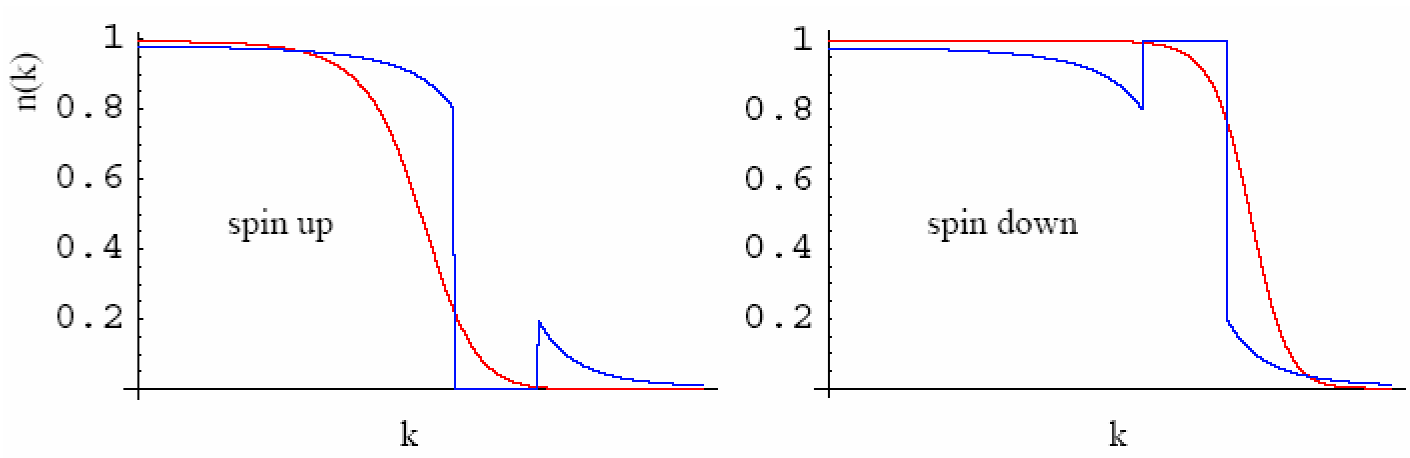
Signature of breached pair superfluidity

(A quantum phase transition from BCS to BP)

BCS



BP

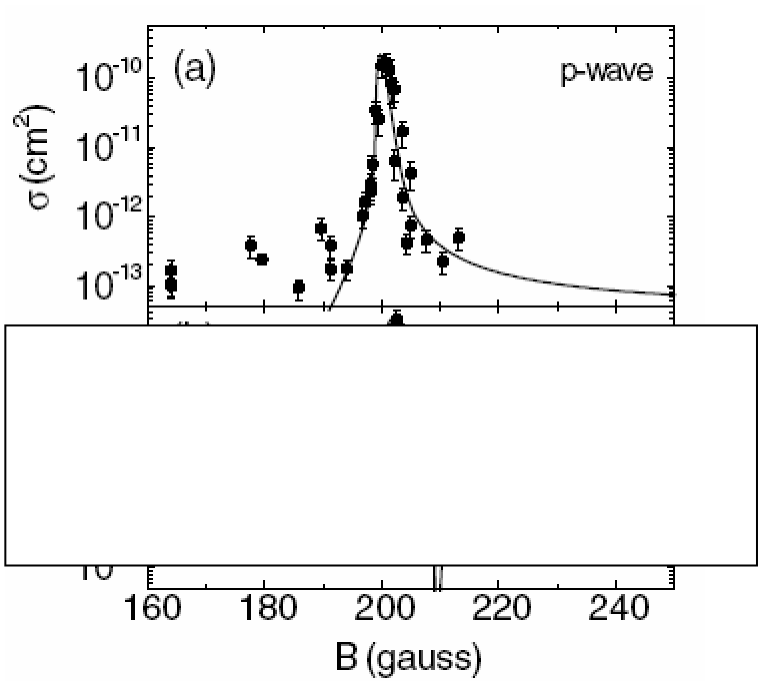


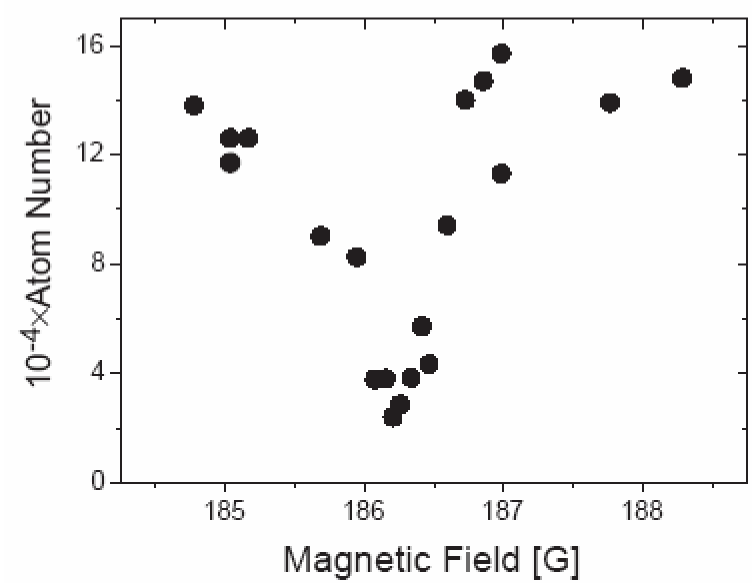
Part 2. Chiral anomaly of an atomic Fermi gas in p-wave resonance

[WVL, submitted for publication,
to be posted cond-mat/0503???]

To discover … a domain wall and an anomalous quantum mass flow on it

p-wave Feshbach experiments





$$^{40}\mathrm{K}\ \text{atoms in}\ |\tfrac{9}{2}, -\tfrac{7}{2}\rangle.$$
$$T_F \sim 1\mu\mathrm{K} \sim 0.01\mathrm{G}\times\mu_B.$$

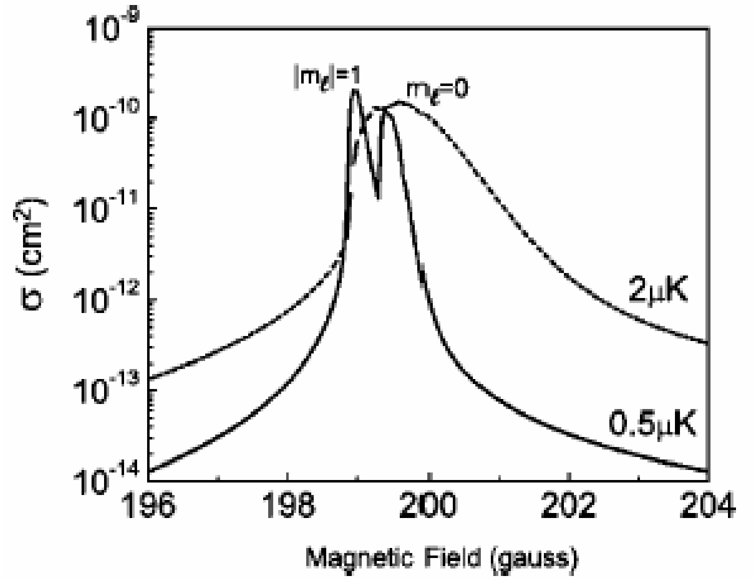
[Regal, Ticknor, Bohn and Jin, PRL (2003)]

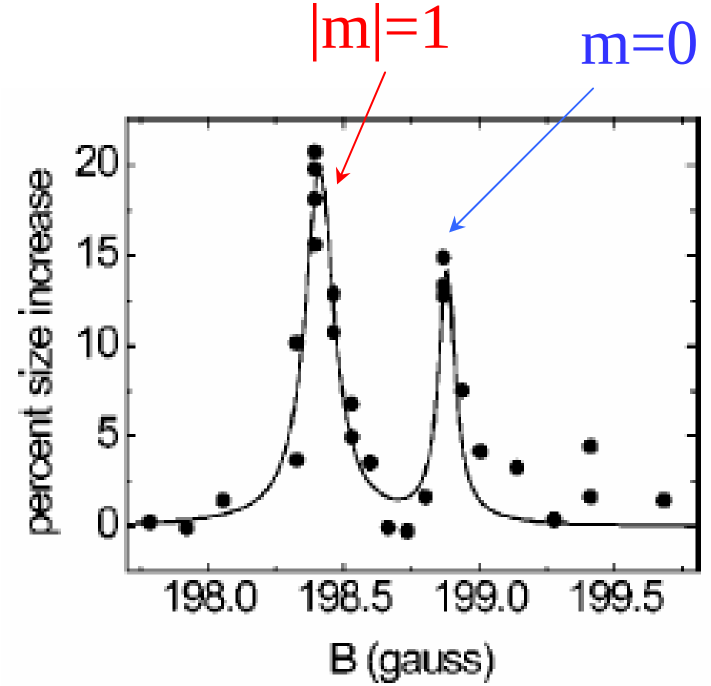
$$^{6}\mathrm{Li}\ \text{in}\ |\tfrac{1}{2}, \tfrac{1}{2}\rangle + |\tfrac{1}{2}, -\tfrac{1}{2}\rangle.$$
$$T_F \sim 10\mu\mathrm{K}\sim 0.1\mathrm{G}\times\mu_B$$

[J.Zhang, C. Salomon, et al., cond-mat/0406085]

Anisotropy in p-wave Feshbach resonances

p-wave: $l = 1, m = -1, 0, +1$





Calculated *Experimentally observed*

[by C. Ticknor, et al., PRA (2004)]

Fermi temperature: $T_F \sim 1\mu\mathrm{K} \sim 0.01\mathrm{G}\cdot\mu_B$

Theoretical modeling

strongly anisotropic interactions;
separate x,y orbitals from z orbital

Planar p-wave model for fermionic atoms

Focus:

On p_x and p_y orbital interactions;
All fermions in a single spin state.

Model:

$$\boxed{H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}} + \frac{g}{2\mathcal{V}} \sum_{\mathbf{q}\mathbf{k}\mathbf{k}'} \vec{k} \cdot \vec{k}' a^{\dagger}_{\frac{\mathbf{q}}{2}+\mathbf{k}} a^{\dagger}_{\frac{\mathbf{q}}{2}-\mathbf{k}} a_{\frac{\mathbf{q}}{2}+\mathbf{k}'} a_{\frac{\mathbf{q}}{2}-\mathbf{k}'}}$$

Notation:

- boldface vector: $\mathbf{k} = (k^x, k^y, k^z)$
- arrow vector: $\vec{k} = (k^x, k^y)$ $=$ planar vector
- $\epsilon_{\mathbf{k}} = \dfrac{\mathbf{k}^2}{2m} - \mu$
- $\mathcal{V}$=space vol.; g=coupling

The order parameter

p-wave pair operator:

$$\vec{\Phi}_{\mathbf{q}} = -\frac{g}{\mathcal{V}} \sum_{\mathbf{k}} \vec{k}\, a_{\frac{\mathbf{q}}{2}-\mathbf{k}} a_{\frac{\mathbf{q}}{2}+\mathbf{k}} \,.$$

Complex, 2-component vector

4 real variables

Parameterization:

$$\langle \vec{\Phi} \rangle \equiv \begin{pmatrix} \langle \Phi^x_{\mathbf{q}=0} \rangle \\ \langle \Phi^y_{\mathbf{q}=0} \rangle \end{pmatrix} = \rho e^{i\vartheta \mathbb{1}} e^{-i\varphi\sigma_2} \begin{pmatrix} \cos\chi \\ i\sin\chi \end{pmatrix},$$

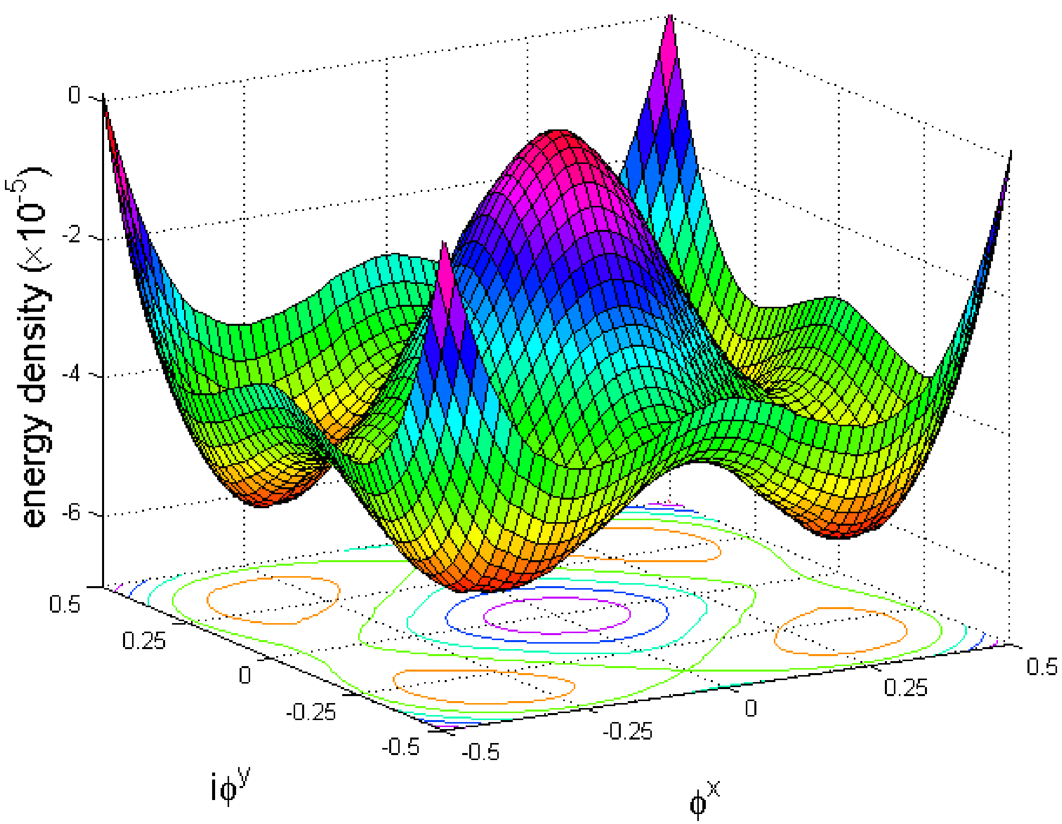
2×2

Pauli matrix

ϑ=overall phase. φ=rotation angle in orbital space

ϑ, φ are Goldstone bosons---gapless collective excitations.

Calculated effective potential (free energy at T=0)



$$\langle\vec{\Phi}\rangle = \rho e^{i\vartheta} e^{i\sigma_2\varphi}\begin{pmatrix}\cos\chi \\ i\sin\chi\end{pmatrix}$$

$$(\text{set } \vartheta = \varphi = 0)$$

Units of :

- ★ $energy = \epsilon_F = \mu$
- ★ $length = k_F^{-1}$

Order space coordination:

$$(\phi^x, \quad i\phi^y) = (\rho\cos\chi, \quad -\rho\sin\chi)$$

Energy minima at:

$$\rho \neq 0$$
$$\chi = \pm\tfrac{\pi}{4}, \ \pi \pm \tfrac{\pi}{4}$$

Analytical expression of the effective potential

$$V[\rho,\chi] = \frac{\rho^2}{2|g|} - \int \frac{d^3\mathbf{k}}{2(2\pi)^3}\left[\sqrt{\epsilon_{\mathbf{k}}^2 + \rho^2(k_x^2\cos^2\chi + k_y^2\sin^2\chi)} - |\epsilon_{\mathbf{k}}|\right]$$

Axial superfluid

$$\langle\vec{\Phi}\rangle = \frac{\rho_0}{\sqrt{2}}\begin{pmatrix}1\\ \pm i\end{pmatrix} \qquad \text{for} \qquad \chi = \pm\frac{\pi}{4}$$

Known as:

In other p-wave models:

$p_x + ip_y$ state; axial state;
Anderson-Brinkman-Morel state (He-3 A phase);

first predicted by Ho and Diener, cond-mat/0408468 and independently by Gurarie, Radzihovsky, Andreev, cond-mat/0410620

Orbital angular mometum (macroscopic):

$$L_z^{\text{total}} = \pm\frac{N_0}{2}\hbar \quad \text{for} \quad \chi = \pm\frac{\pi}{4}\,.$$

$N_0 =$ of atoms in condensed pairs

Effective field theory of the order parameter field

$$\langle \vec{\Phi}(\mathbf{r}) \rangle = \begin{pmatrix} \langle \Phi^x(\mathbf{r}) \rangle \\ \langle \Phi^y(\mathbf{r}) \rangle \end{pmatrix} = \rho_0 \begin{pmatrix} \cos\chi(\mathbf{r}) \\ i\sin\chi(\mathbf{r}) \end{pmatrix}$$

(spatially nonuniform)

(ρ_0=constant)

Free energy functional

$$F[\chi] = \frac{\rho_0^2}{C_\phi} \int d^3\mathbf{r} \left[\frac{1}{2} (\nabla \chi)^2 + \frac{1}{4\xi^2} \Big(1 + \cos(4\chi) \Big) \right]$$

[sine-Gordon theory in 3D Euclidean space]

minimized by $\chi_\pm = \pm\frac{\pi}{4}$

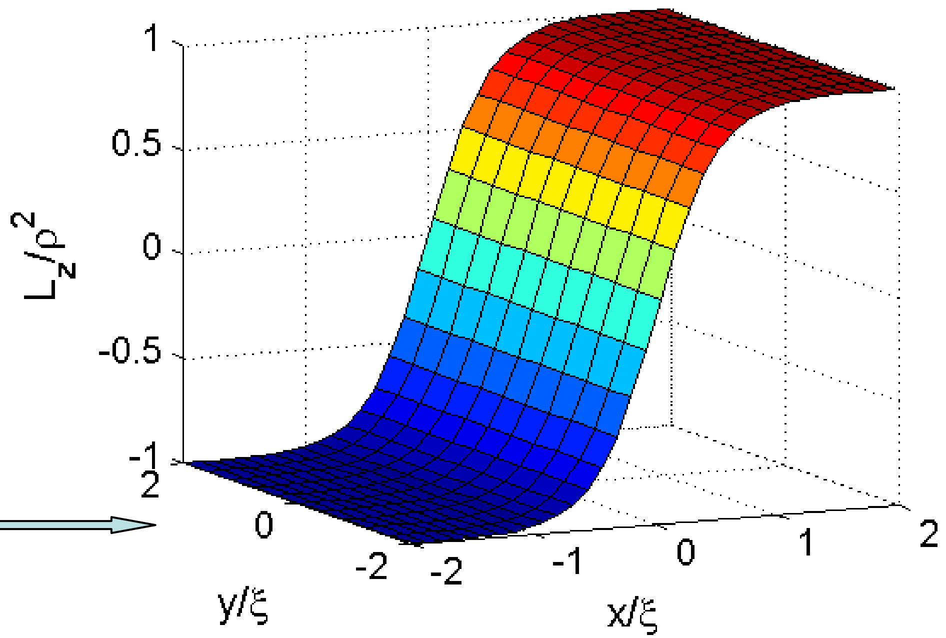
Physical understandings:

1. $\rho_0 = \Delta_0 / k_F$; Δ_0=maximum energy gap
2. $C_\phi \simeq 1/(mk_F)$, nonuniversal (renormalizable), related to the mass of p-wave pairs (molecules).
3. $\xi \simeq \Delta_0 / v_F$=coherence length

Domain walls as topological defects ---solitons

Solution to field equation

$$\chi_{\pm}(\mathbf{r}) = \pm\arctan\Big(\tanh(x/\xi)\Big)$$



$\chi = +\frac{\pi}{4}$

$\chi = -\frac{\pi}{4}$

$$\frac{L_z}{\rho^2} = \sin 2\chi$$

use $\vec{L} = -i\langle\vec{\Phi}\rangle^* \times \langle\vec{\Phi}\rangle$

domain wall energy per unit area $= \Delta_0^2/(2C_\phi \xi k_F^2) \sim \Delta_0/\xi^2$

Quantum anomaly

[chiral mass flow of atoms in the groundstate]

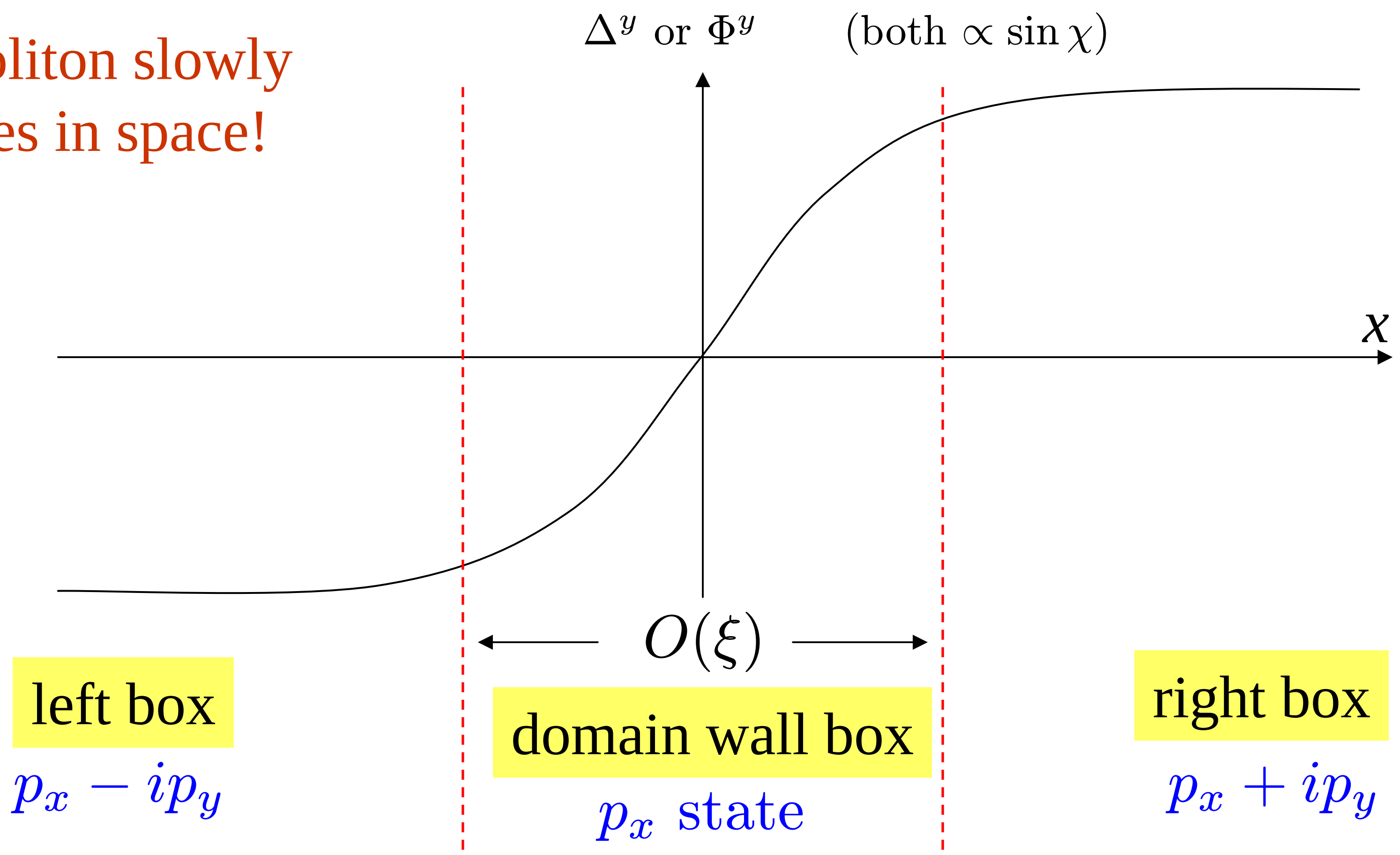
Heuristic steps to discover the anomaly

. . .

Anomaly (II)

View of three macro-boxes

A soliton slowly varies in space!

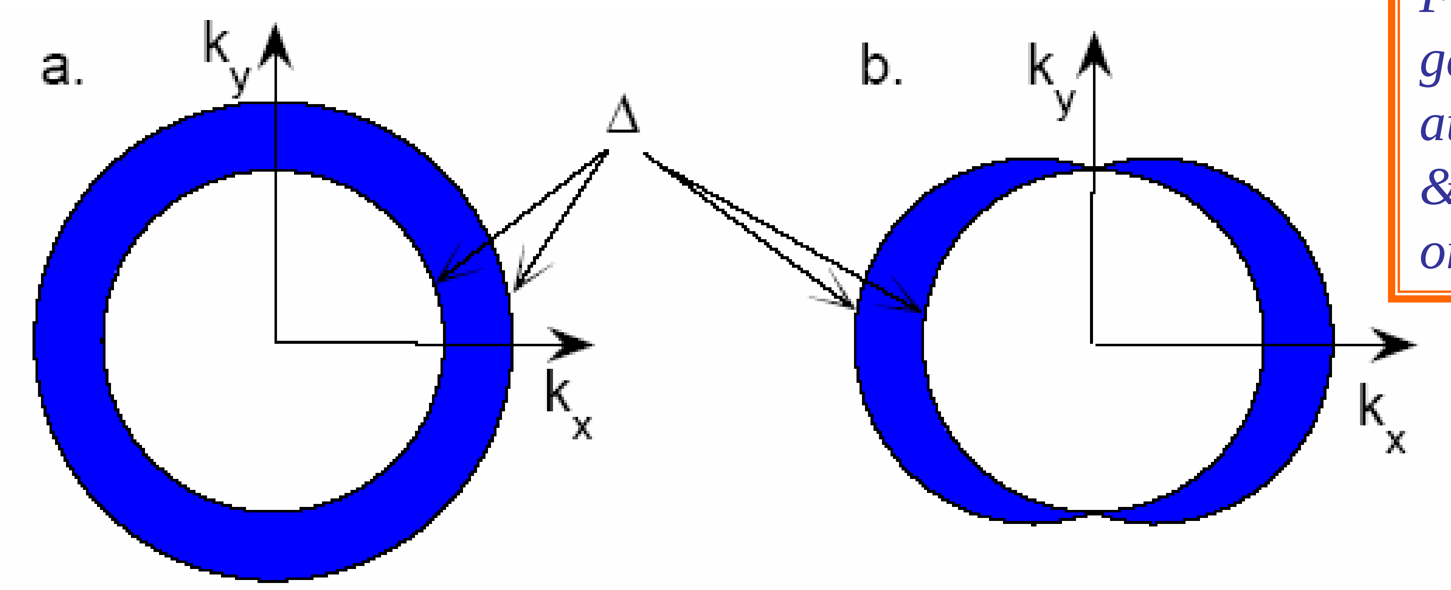


Important lengths $\xi \gg k_F^{-1}$, $\quad (k_F^{-1} \sim 1/\sqrt[3]{\text{density}})$

Anomaly (III)

The energy gap of quasiparticles (fermionic, atomic states)

$p_x + ip_y$ state $\qquad$ p_x state

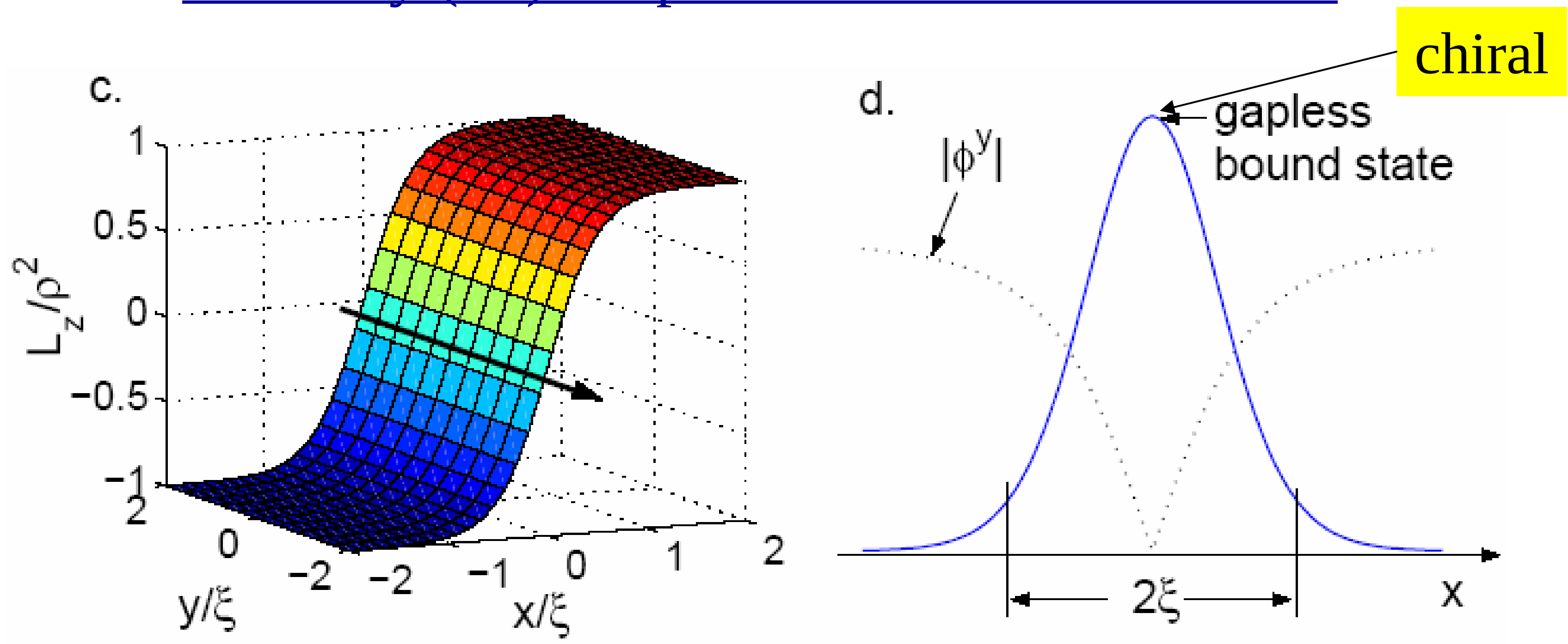


For both states, gap vanishes at the north & south poles on z-axis!

spectrum: $$E_{\mathbf{k}} = \sqrt{\left(\frac{\mathbf{k}^2}{2m} - \mu\right)^2 + \rho_0^2 (\underbrace{k_x^2}_{\Delta^x} \cos^2\chi + \underbrace{k_y^2}_{\Delta^y} \sin^2\chi)}$$

Representative point: $\mathbf{k} = (0, k_F, 0)$

Anomaly (IV): Gapless fermion bound states



Solve Bogoliubov-de Gennes equations *(similar to Jakiew-Rebbi problem of Dirac fermions)*

$$E = \pm\epsilon_{\mathbf{k}_\parallel} \text{ for } k_y \geq 0 \text{ or } \leq 0$$
$$\text{with } \mathbf{k}_\parallel = (0, k_y, k_z)$$

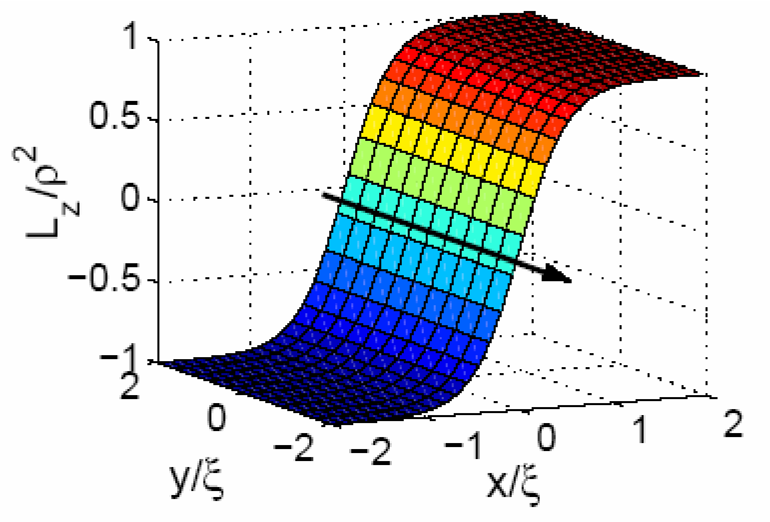
⇨ found bound states analytically for $\xi \gg k_F^{-1}$

Anomalous quantum mass flow of atoms in the groundstate (No additional external force!)

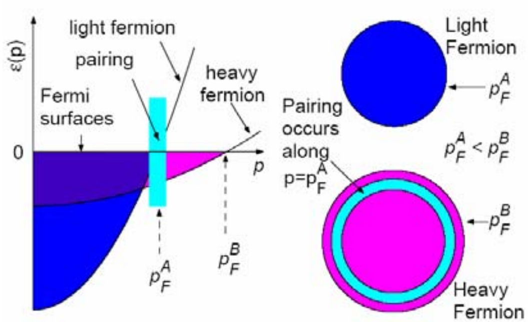
mass current per unit *z*-length:

$$\mathbf{j} = -\frac{\hbar k_F^3}{6\pi^2}\hat{e}_y \, ,$$

$(\hbar \text{ restored for clarity})$



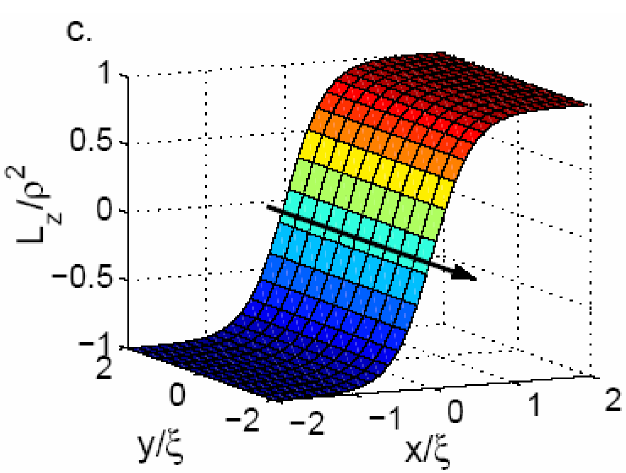
Summary

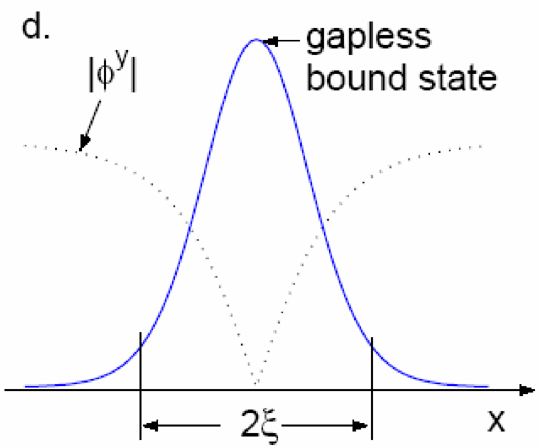


Breached Pair Superfluidity

[courtesy of Phys. Rev. Focus (Jan 2005)]

Domain wall quasiparticles and chiral anomaly in p-wave





[submitted for publication]