# Breached pair superfluidity

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publications

1. PRL **90**, 047002 (2003)
2. PRL **91**, 032001(2003)
3. PRA **70,** 033603 (2004)
4. PRL **94**, 017001 (2005)

# Breached pair superfluidity (BP)

News story: “Odd particle out”,
Phys. Rev. Focus (January 5, 2005; story 1)

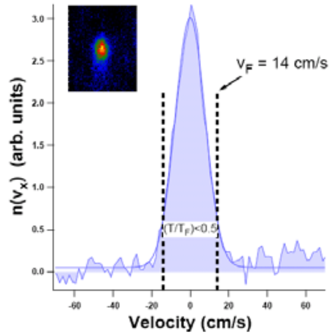
# Outline

1. Motivation for new pairing
2. Heuristics of breached pair superfluidity (BP state)
3. Stability issue
4. Comment on recent developments: Strong-coupling BP
5. Experimental realization

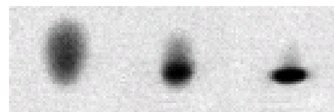
**Part 1.**

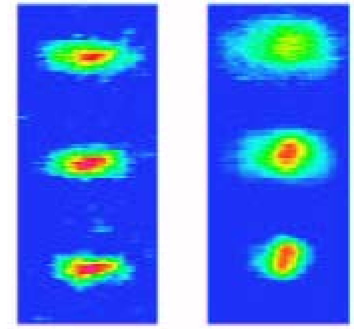
**Motivation for new pairing**

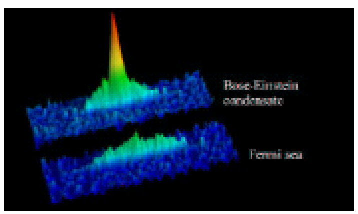
# Current: Superfluidity in atomic Fermi gases of $^{6}$Li, $^{40}$K, . . .



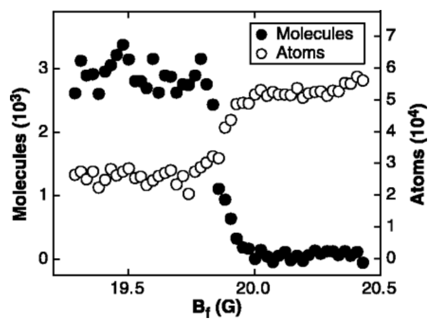
J. Thomas (Duke)


W. Ketterle (MIT)

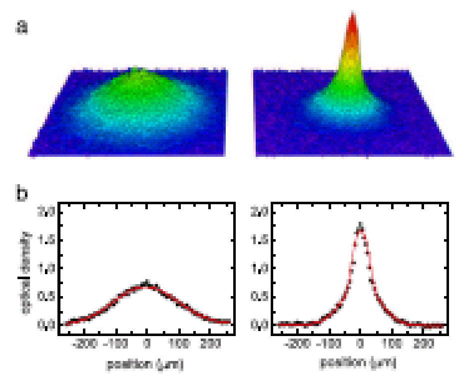

M. Inguscio (Firenze)



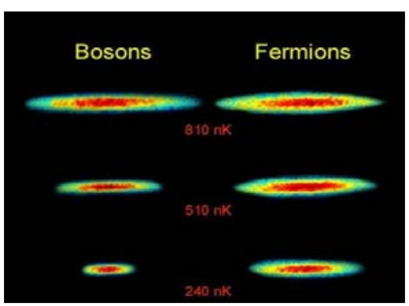
C. Salomon (ENS, Paris)



R. Grimm (Innsbruck)



D. Jin (JILA)



R. Hulet (Rice)

# Motivation: atomic Fermi gases

- BCS superfluidity of fermionic atoms. charge neutral; highly tunable; high Tc superfluidity (speculated by Demler, et al); …
- BEC of molecules, BEC/BCS crossover superfluids, pseudogap (relevant to high Tc superconductivity?), …

***Pairing with mismatched fermi surfaces***

- The two spin components can have *density imbalance*
- FFLO [Larkin and Ovchinnikov; Fulde and Ferrell, 1964]: indirect evidence in heavy fermions (CeCoIn5?), ...
- new pairing possibility? — “breached pairing”

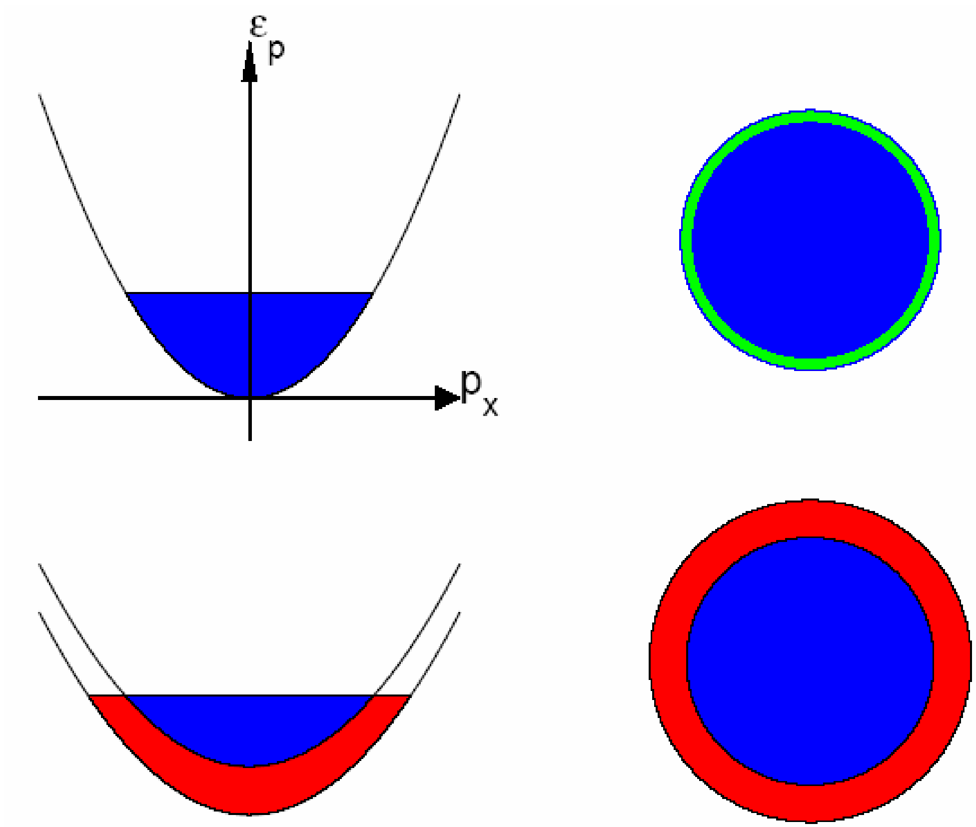
## Part 2.
## Heuristics of Breached Pair Superfluidity
## (BP state)

# Different kinds of pairing

| | |
|---|---|
| BCS | Bardeen-Cooper-Schrieffer (1957) |
| FFLO | Fulde and Ferrell (1964); independently Larkin and Ovchinnikov (1965) |
| Breached pairing | Pairing occurs within the interior or exterior of a large Fermi ball. [This talk!]. |

# Heuristic introduction to BP

*Recall BCS pairing*



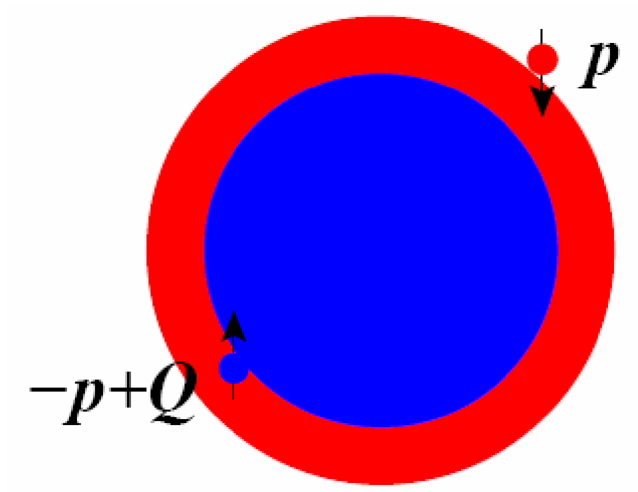

momentum gap

$$\kappa = \frac{\Delta}{v_F}$$

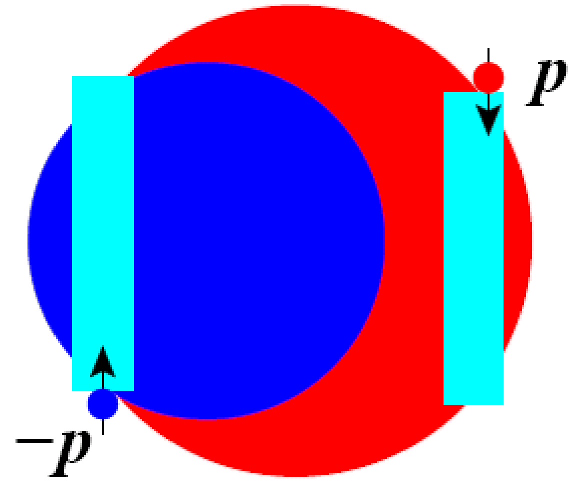
$$\delta p_F \equiv p_F^{\downarrow} - p_F^{\uparrow}$$

when $\delta p_F \gtrsim \kappa$,
BCS impossible!

# *Recall FFLO paring*




$$\epsilon_\uparrow(\mathbf{p}), \epsilon_\downarrow(\mathbf{p})$$




$$\epsilon_\uparrow(\mathbf{p}+\mathbf{Q}), \epsilon_\downarrow(\mathbf{p})$$

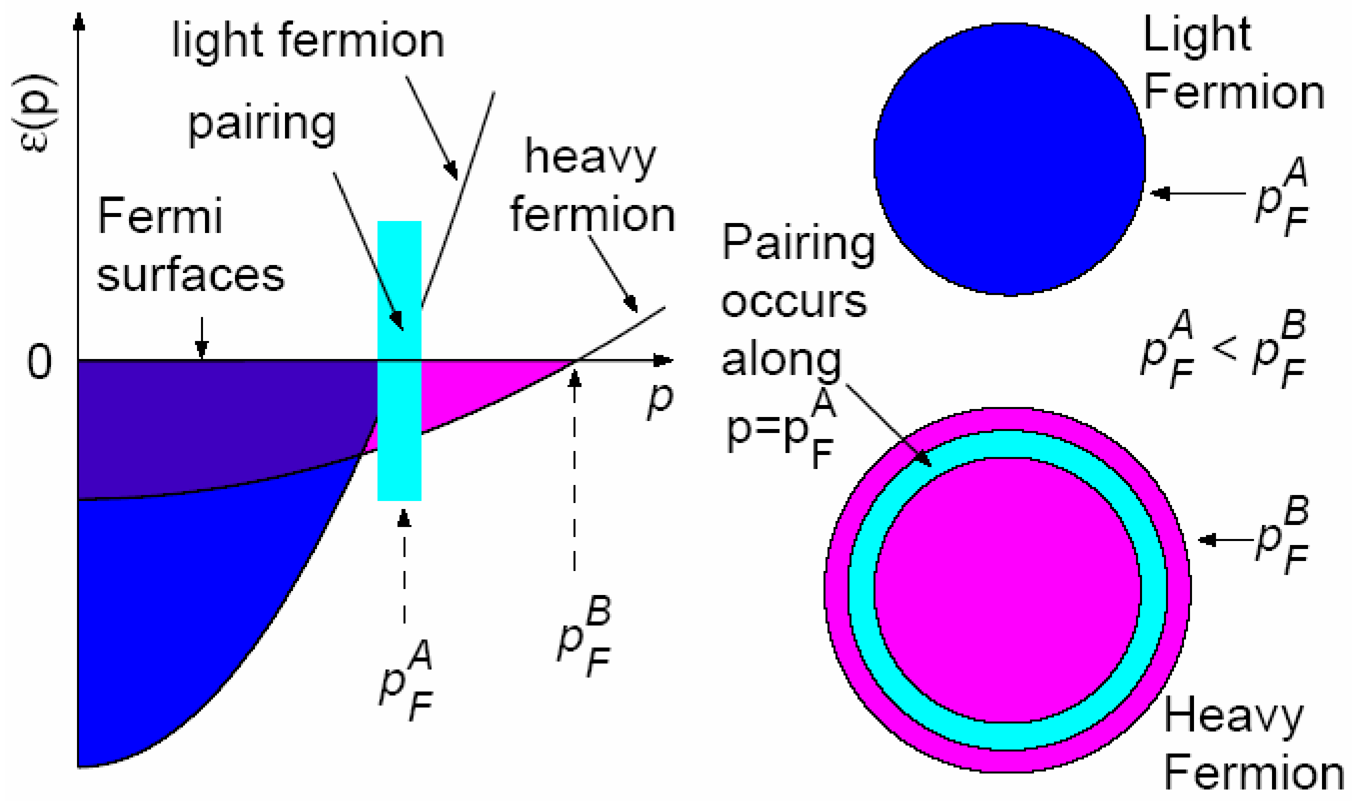
FFLO superconducting state:

- ★ momentum space: $\langle \psi_{\mathbf{p}+\mathbf{Q}\uparrow} \psi_{-\mathbf{p}\downarrow} \rangle \neq 0$
- ★ modulation in position space: $\langle \psi_\uparrow(\mathbf{x}) \psi_\downarrow(0) \rangle \sim e^{i\mathbf{Q}\cdot\mathbf{x}}, \cos(\mathbf{Q}\cdot\mathbf{x}), \ldots$

For either BCS or LOFF: $T_c \sim E_F \, e^{-\frac{1}{|g|N(0)}}$

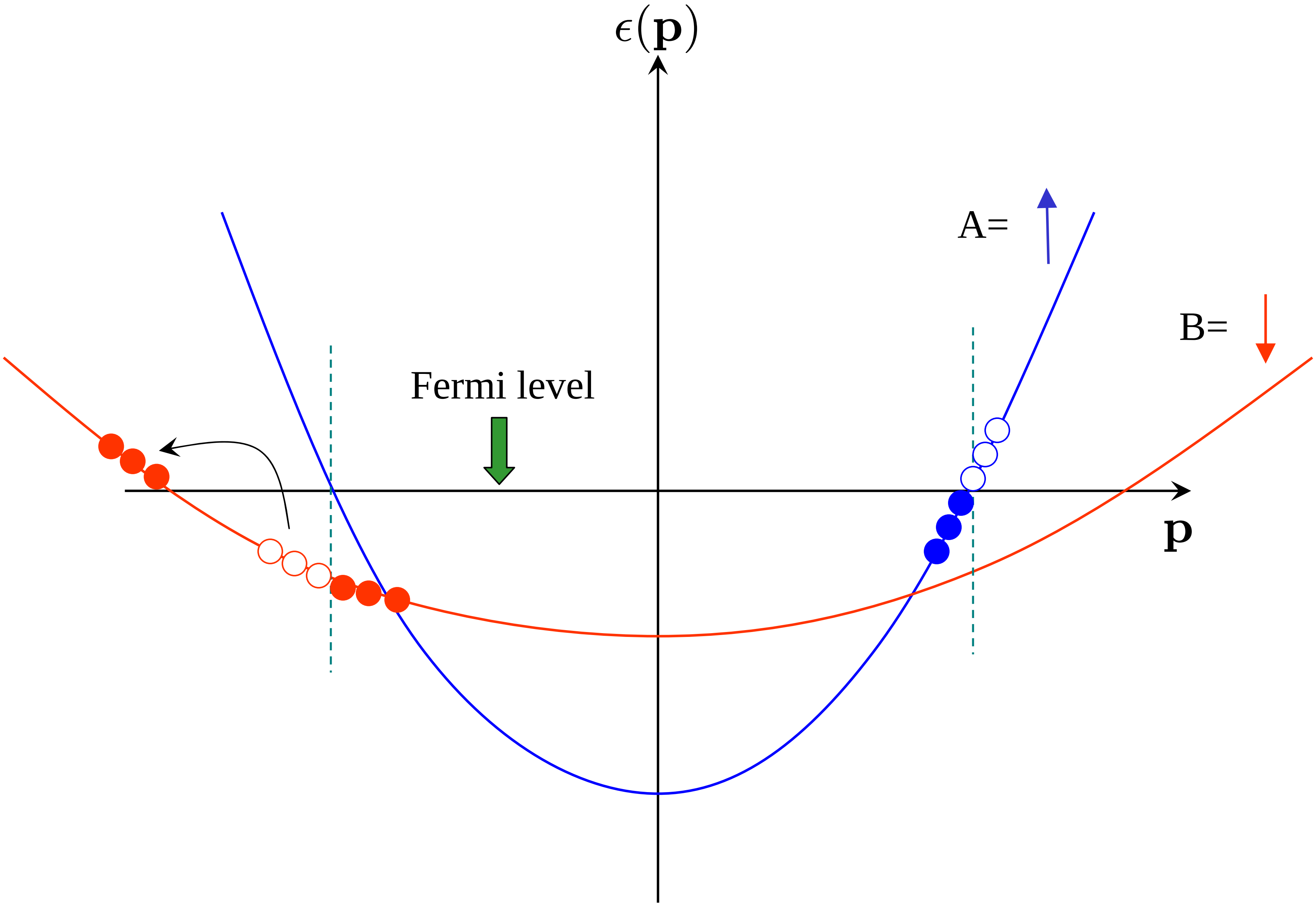
coupling → $|g|$; density-of-state → $N(0)$

# *Breached Pair Superfluidity (BP)*




[WVL, F. Wilczek, PRL (2003)]

BP state = a superfluid + a normal Fermi liquid at T=0;
has gapped and gapless quasiparticle excitations.


$\epsilon(\mathbf{p})$
A=
B=
Fermi level
$\mathbf{p}$

# Heuristic of Breached Pair

For a *momentum* gap of order $\kappa$:

To gain condensation energy per pair:

$$\epsilon_{\rm pair} = \kappa \left( \frac{p_F^\uparrow}{m_\uparrow} + \frac{p_F^\uparrow}{m_\downarrow} \right) > \frac{{p_F^\downarrow}^2 - {p_F^\uparrow}^2}{2m_\downarrow}$$

To realize a breach:

$$p_F^\downarrow - p_F^\uparrow > \kappa$$

Two conditions compatible if:

$$1 > \frac{p_F^\downarrow + p_F^\uparrow}{2p_F^\uparrow} \frac{m_\uparrow}{m_\uparrow + m_\downarrow}.$$

***Remark***

The consistency condition can be satisfied for arbitrarily small $\kappa$ (weak coupling) when $m_\downarrow \gg m_\uparrow$.

# Weak coupling BP

Model:

$$H = \sum_{\mathbf{p}} \epsilon_{\mathbf{p}\alpha} \psi^\dagger_{\mathbf{p}\alpha} \psi_{\mathbf{p}\alpha} + \sum_{\mathbf{p}\mathbf{p}'} V(\mathbf{p}-\mathbf{p}') \psi^\dagger_{\mathbf{p}\uparrow} \psi^\dagger_{-\mathbf{p}\downarrow} \psi_{-\mathbf{p}'\downarrow} \psi_{\mathbf{p}'\uparrow}$$

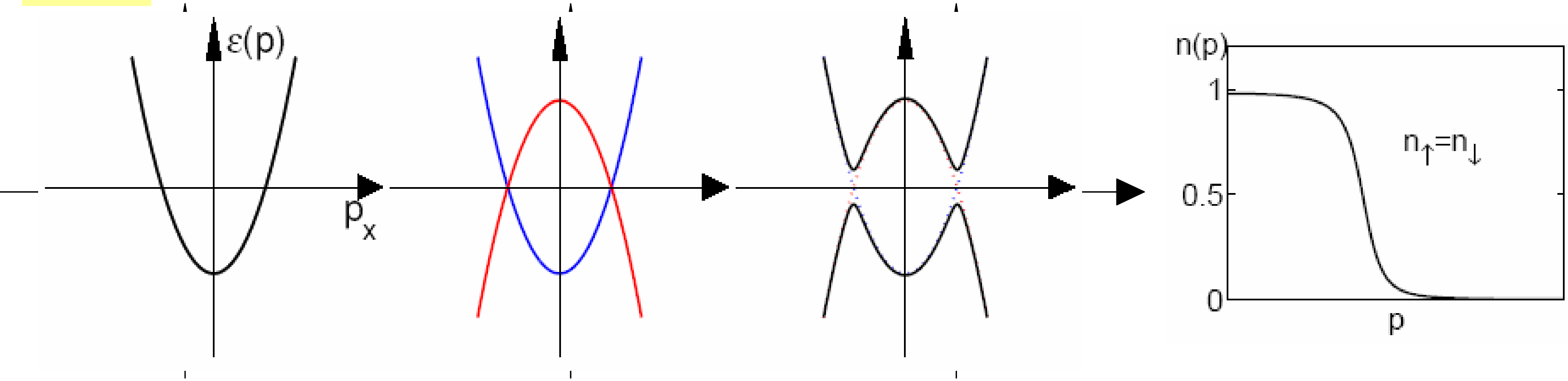
$$\epsilon_{\mathbf{p}\alpha} = \frac{\mathbf{p}^2}{2m_\alpha} - \mu_\alpha \,, \qquad \alpha = \uparrow, \downarrow$$

For real-space
δ-like interaction

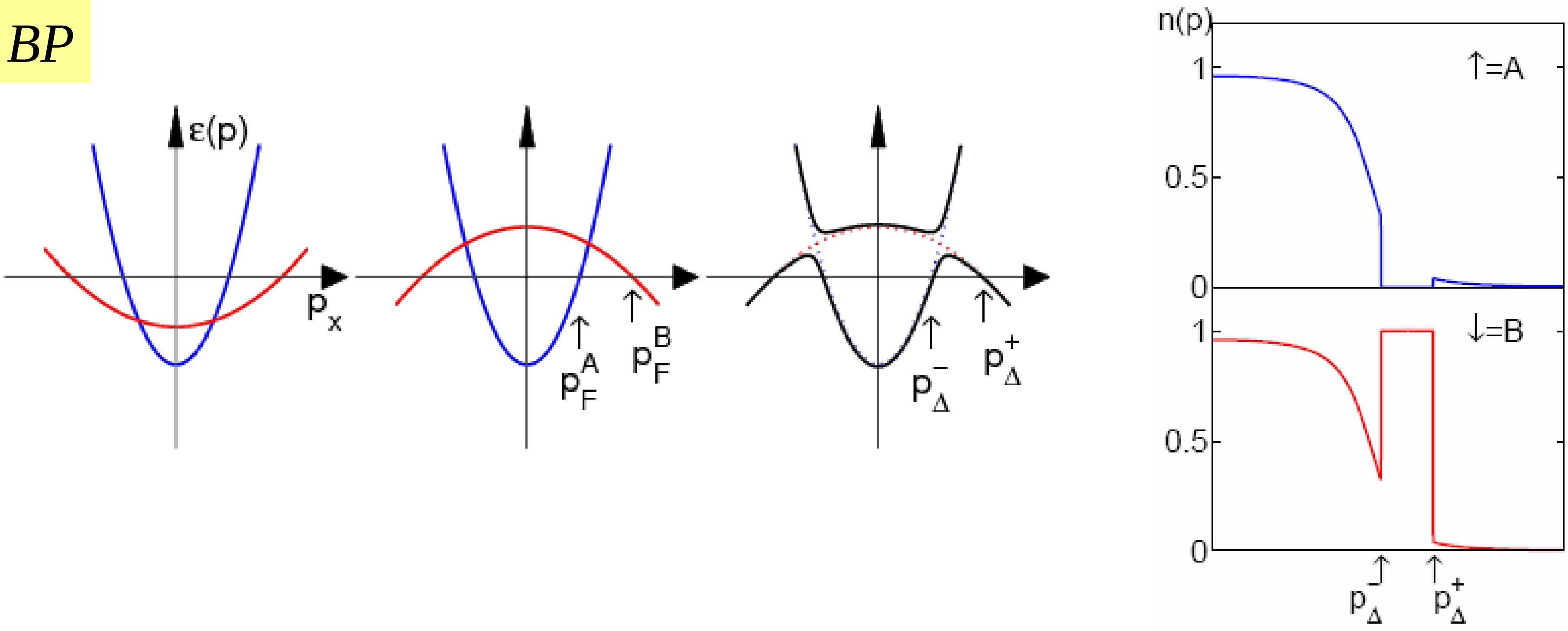
$$\Longrightarrow \quad V(\mathbf{q}) = g = \frac{4\pi\hbar^2 a_s}{m} = const$$

$$(a_s < 0)$$

*BCS*




*BP*



# *Many body wavefunction*

## BCS vs BP

Unpaired matter??

$$|BCS\rangle = \prod_{\mathbf{p}} (u_{\mathbf{p}} + v_{\mathbf{p}} \psi^{\dagger}_{\mathbf{p}\uparrow} \psi^{\dagger}_{-\mathbf{p}\downarrow})|0\rangle$$

$$|BP\rangle = \prod_{\mathbf{p}\notin \mathrm{breach}} (u_{\mathbf{p}} + v_{\mathbf{p}} \psi^{\dagger}_{\mathbf{p}\uparrow} \psi^{\dagger}_{-\mathbf{p}\downarrow}) \prod_{\mathbf{p}\in \mathrm{breach}} \psi^{\dagger}_{\mathbf{p}\downarrow} |0\rangle$$

where

$$\begin{pmatrix} u^2_{\mathbf{p}} \\ v^2_{\mathbf{p}} \end{pmatrix} = \frac{1}{2}\left(1 \pm \frac{\epsilon^{+}_{\mathbf{p}}}{\sqrt{{\epsilon^{+}_{\mathbf{p}}}^2 + \Delta^2_{\mathbf{p}}}}\right) \qquad \epsilon^{\pm}_{\mathbf{p}} \equiv \frac{\epsilon_{\mathbf{p}\uparrow} \pm \epsilon_{\mathbf{p}\downarrow}}{2}$$

"breach" region: $p^{-}_{\Delta} \le |\mathbf{p}| \le p^{+}_{\Delta}$

## *Many body wavefunction (continued)*

"breach" region: $p_{\Delta}^{-} \leq |\mathbf{p}| \leq p_{\Delta}^{+}$ *by* $E_{\mathbf{p}}^{+} E_{\mathbf{p}}^{-} = 0 \Rightarrow p_{\Delta}^{\pm}$

QP spectrum: $E_{\mathbf{p}}^{\pm} = \epsilon_{\mathbf{p}}^{-} \pm \sqrt{{\epsilon_{\mathbf{p}}^{+}}^{2} + \Delta_{\mathbf{p}}^{2}}$.

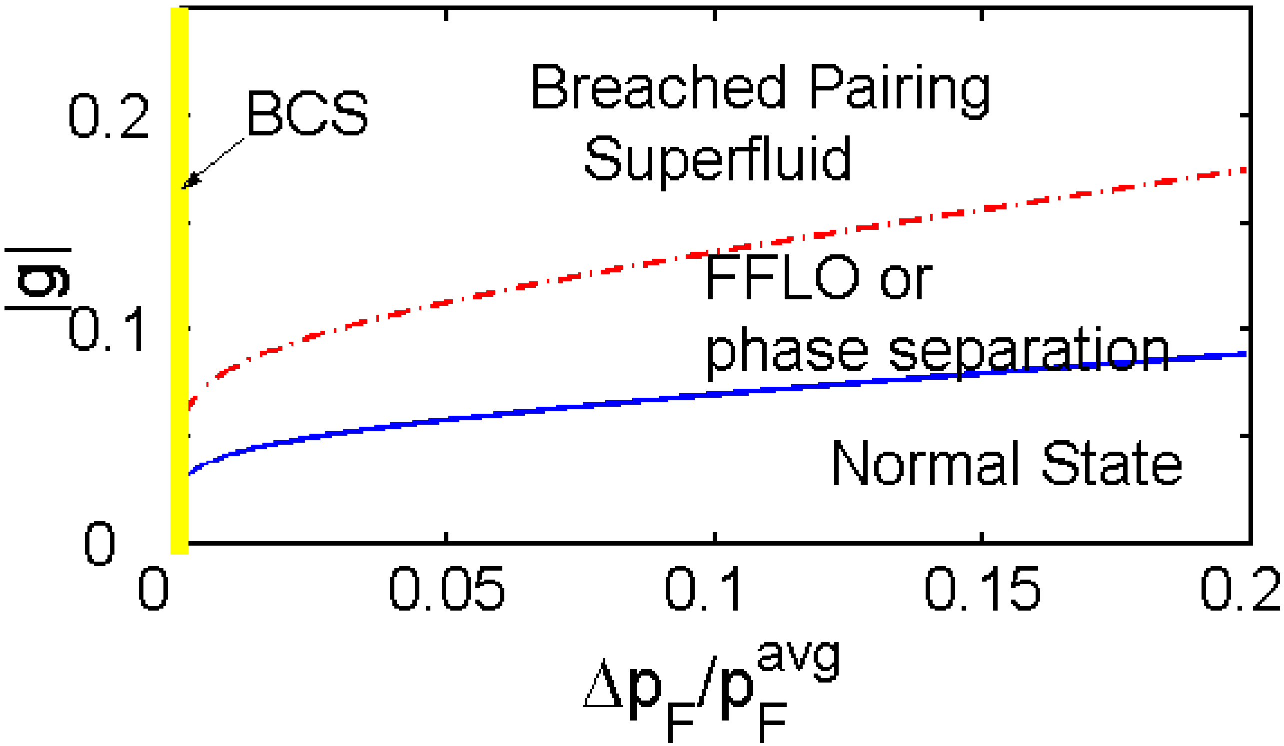
# Important features of breached pair (BP) state

## Feature summary

1. a new kind of pairing;
2. coexisting superfluid and normal Fermi liquid components at T=0 (quantum state);
3. gapped and gapless quasiparticles
4. does not spontaneously break the translational and rotational symmetries
5. momentum-space phase separation

# Phase diagram (fixing two densities separately)

$$V(\mathbf{p}-\mathbf{p}) = \begin{cases} g & \text{if both } \mathbf{p},\mathbf{p}' \text{ near fermi surface} \\ 0 & \text{otherwise} \end{cases}$$




Parameters:

$$m_\uparrow/m_\downarrow = 7$$

$$\Delta p_F \equiv p_F^\uparrow - p_F^\downarrow$$

[adapted and modified from WVL and F. Wilczek, PRL (2003)]

obtained by variational method

# Part 3.

# Stability of BP

# How stable?

**The stability of BP criticized by:**

1. Shin-Tza Wu, Sungkit Yip, PRA (2003)
2. P. F. Bedaque, H. Caldas, G. Rupak, PRL (2003); Caldas, PRA (2004)

Both are correct, but are done for a short-range delta-interaction.

**The stability issue was clarified in:**

our latest [Forbes, et al, PRL 94, 017001 (2005)]

*Need*

1. *a finite or long range interaction; or*
2. *a momentum cutoff*

$$R^* \gtrsim k_F^{-1}$$

effective range ($R^*$) — inter-atom distance ($k_F^{-1}$)

# Thermodynamic stability

★ Work in a grand canonical ensemble

★ Find the minima of $\Omega(T, V, \mu_\uparrow, \mu_\downarrow, \Delta)$: $\frac{\partial \Omega}{\partial \Delta} = 0$, $\frac{\partial^2 \Omega}{\partial \Delta^2} > 0$.

★ Garantee $\frac{\partial n_{\mathrm{total}}}{\partial \mu_+} > 0$; $\frac{\partial n_{\mathrm{diff}}}{\partial \mu_-} > 0$. [susceptibilities]
where $\mu_\pm = \frac{\mu_\uparrow - \mu_\downarrow}{2}$

★ specify the system by chemical potential instead of densities; real-space phase separation is automatically taken care.

"… the condition for $\rho_s > 0$ [superfluid density] is actually a slightly weaker requirement than the positive susceptibility …"

[Pao, Wu, and Yip, cond-mat/0506437]

# Two models in Grand Canonical Ensemble

$$H = \sum_{\mathbf{p}\alpha} \epsilon_{\mathbf{p}\alpha} \psi^\dagger_{\mathbf{p}\alpha} \psi_{\mathbf{p}\alpha} + \sum_{\mathbf{p}\mathbf{p}'} V(\mathbf{p}-\mathbf{p}') \psi^\dagger_{\mathbf{p}\uparrow} \psi^\dagger_{-\mathbf{p}\downarrow} \psi_{-\mathbf{p}'\downarrow} \psi_{\mathbf{p}'\uparrow}$$

Momentum dependent interactions:

A. a separable potential with “hard” cutoff

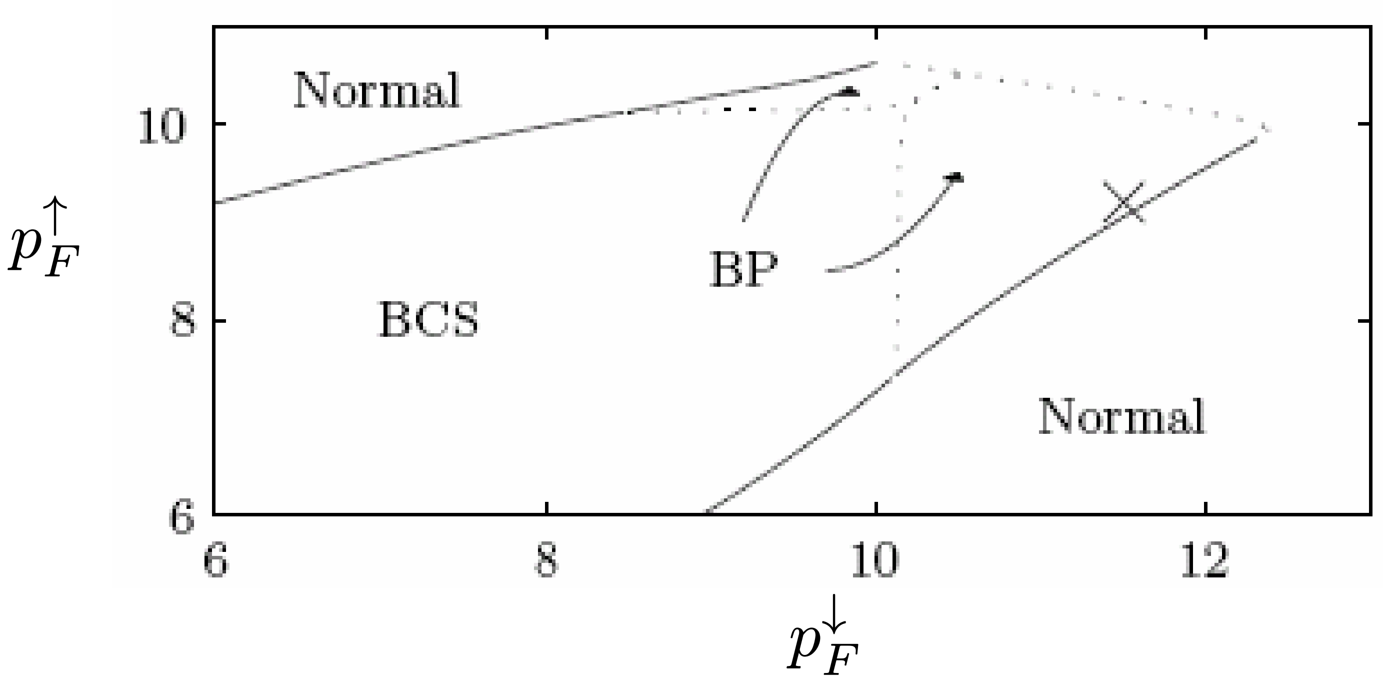
B. a smooth Gaussian

Gap equation for both models:

$$\Delta_{\mathbf{p}} = -\frac{1}{2} \sum_{\mathbf{k}\notin \text{breach}} \frac{|V(\mathbf{p}-\mathbf{k})| \Delta_{\mathbf{k}}}{\sqrt{{\epsilon^+_{\mathbf{k}}}^2 + \Delta^2_{\mathbf{k}}}}$$

# Model A: Interaction with "hard" cutoff

$$V(\mathbf{p}-\mathbf{p}') \sim \theta(\Lambda - |\mathbf{p}|)\theta(\Lambda - |\mathbf{p}'|)$$



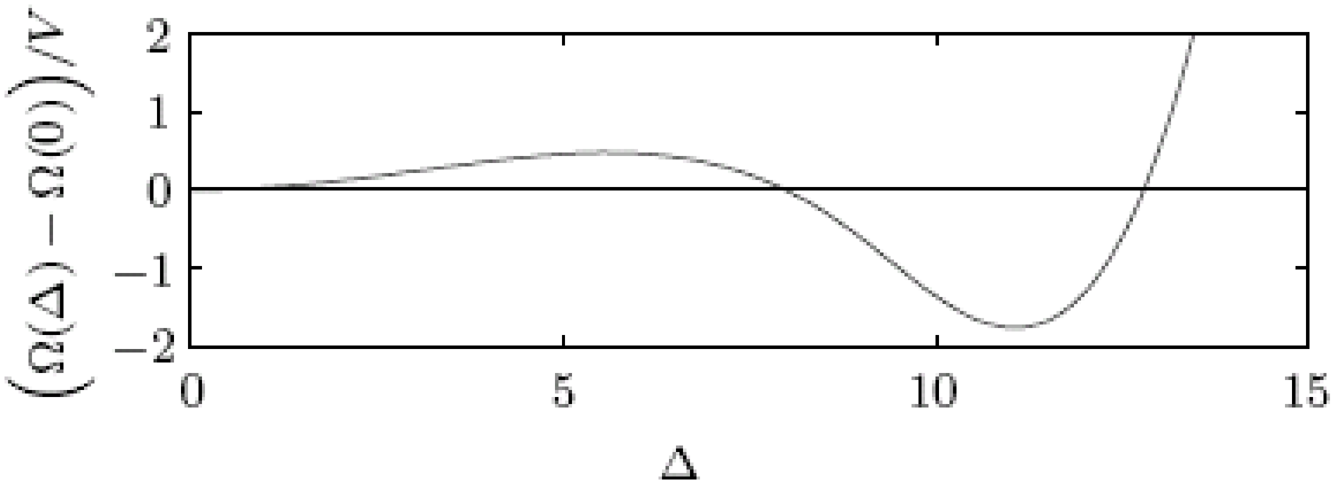

★ $p_F^\alpha \equiv \sqrt{2m_\alpha \mu_\alpha}$.

★ momentum cutoff: $\Lambda$

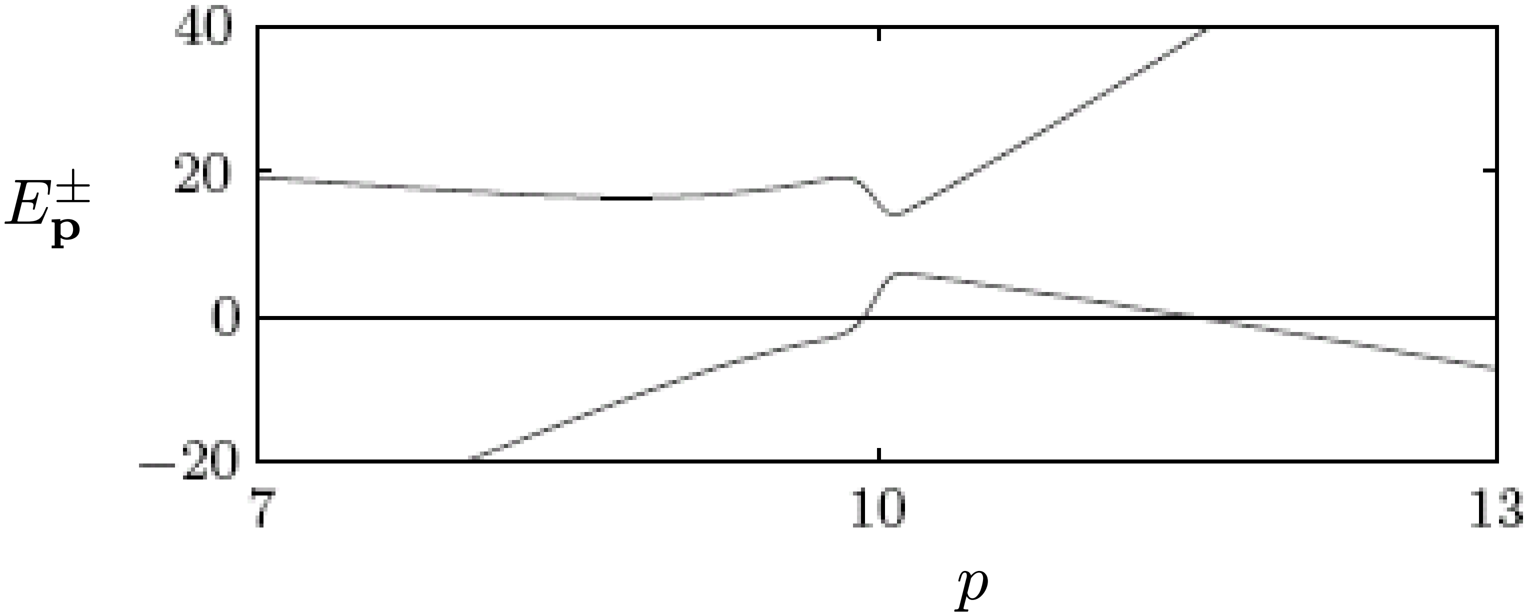
★ momentum unit $\equiv 0.1\Lambda$

[Forbes, Gubankova, WVL, and Wilczek, PRL (2005)]

# Grand thermodynamic potential

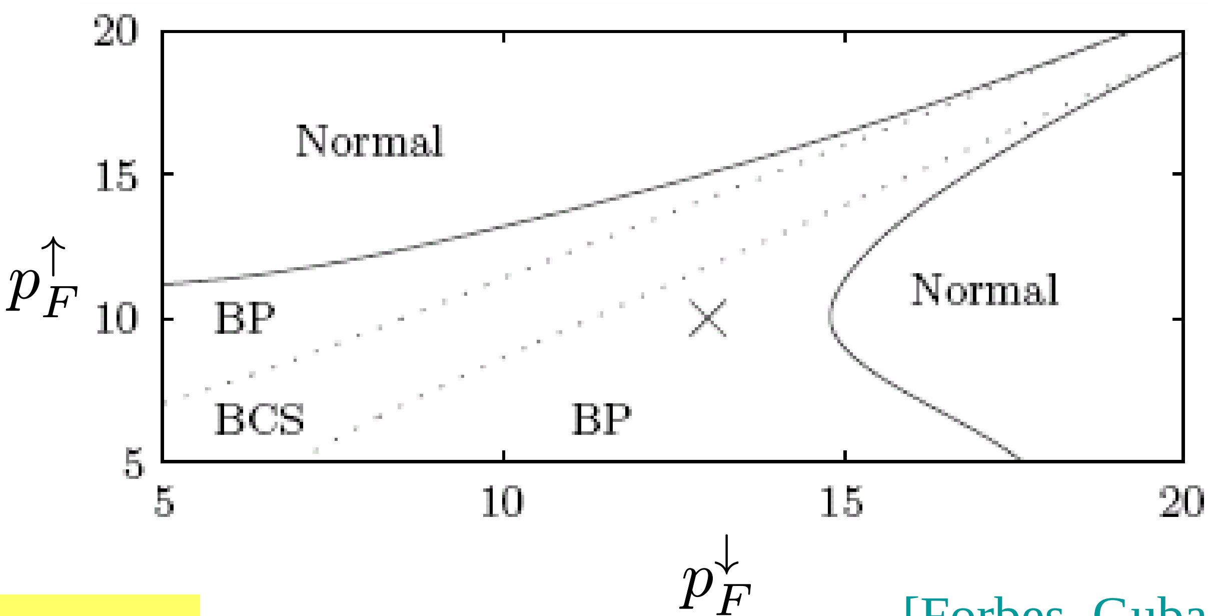


# Quasiparticles in Model A



# Phase diagram: Model B

*Gaussian interaction potential* $V(\mathbf{r}) \sim \exp\left(\frac{-\mathbf{r}^2}{2\lambda^2}\right)$

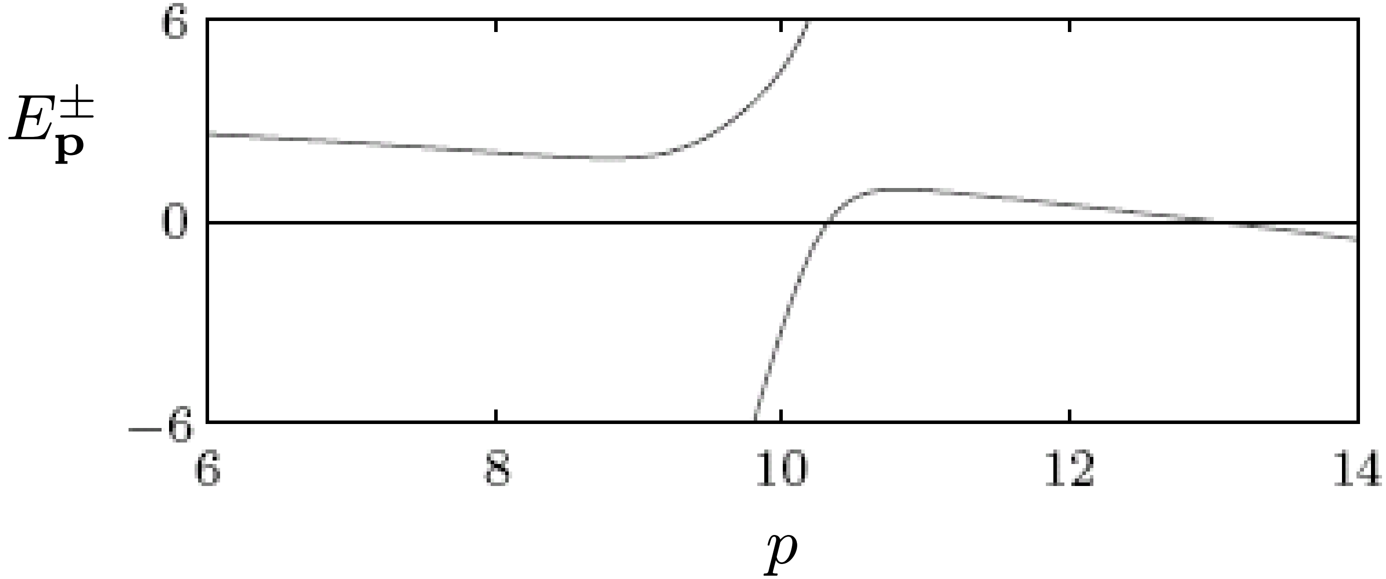



[Forbes, Gubankova, WVL, and Wilczek, PRL (2005)]

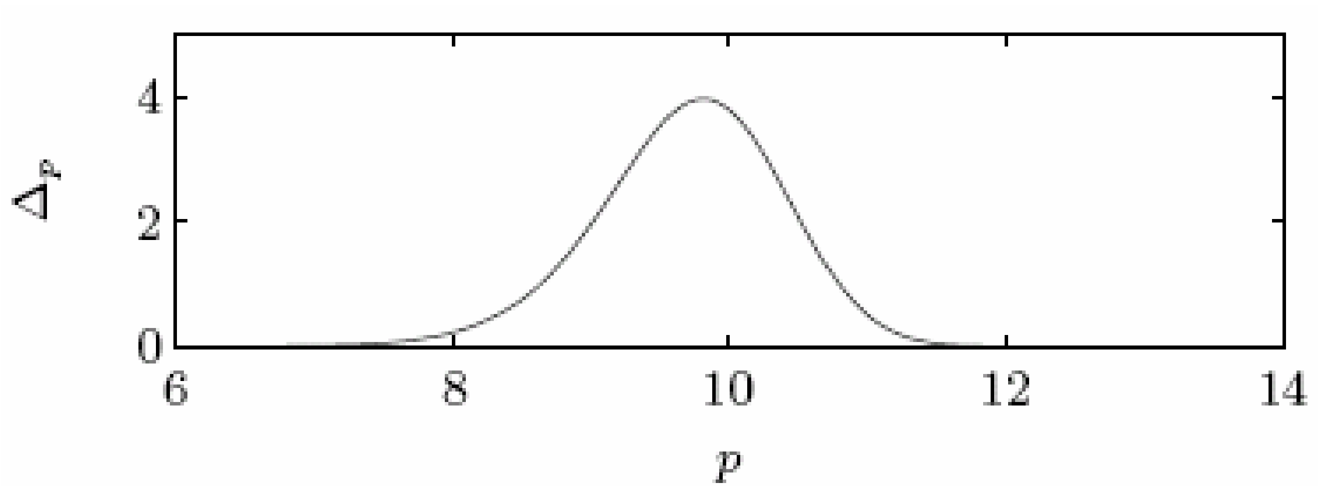
Parameters:

- ★ $p_F^{\alpha} \equiv \sqrt{2m_\alpha \mu_\alpha}.$
- ★ energy units: $\frac{\hbar^2}{m_+\lambda^2} \equiv 1.$
- ★ momentum unit $= \frac{\hbar}{\lambda}.$

# QPs for Model B



# Momentum dependence of energy gap




**Note:** $\Delta_{\mathbf{p}} \neq \text{Constant};$

This is very important in obtaining a positive superfluid density for *a weak-coupling BP*.
[Forbes, WVL, Wilczek, unpublished]

## Part. 4

## Recent development: strong coupling BP

# Case of strong coupling, short-range interaction, and equal mass

- Quantum Monte Carlo [J. Carlson, S. Reddy, PRL (2005)]
- Mean field theories
  - Single-channel resonance model [Pao, Wu, and Yip, cond-mat/0506437]
  - two channel (atom-molecule) model [D. Sheehy, L. Radzihovsky, cond-mat/0508430]
- Effective field theory [D. Son, M. Stephenov, cond-mat/0507586**]**

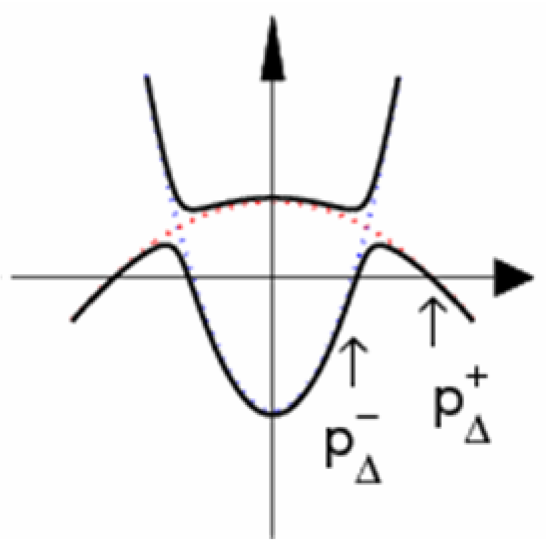
A homogeneous, spin-polarized gapless superfluid [that is a BP] is favored against phase separation in real space.

# BP states of one or two fermi surfaces

**weak coupling**

$$\mu(\Delta) \approx \mu(\Delta = 0) > 0$$

$$m_\uparrow < m_\downarrow$$

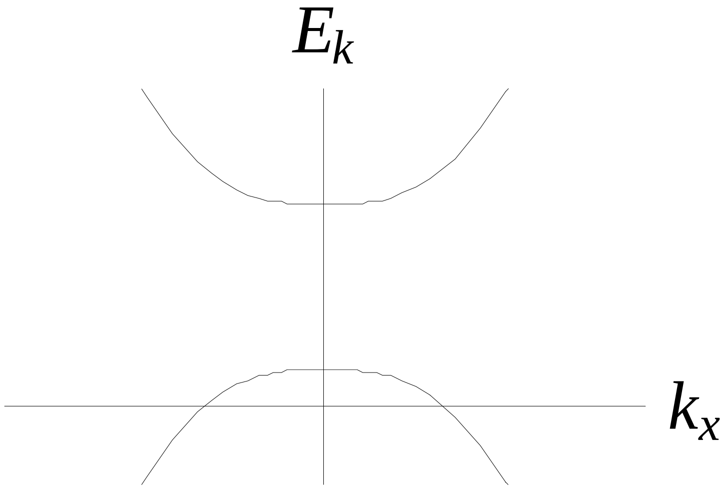



two fermi surfaces

**strong coupling**

$$\mu(\Delta \sim \epsilon_F) < 0$$

$$m_\uparrow = m_\downarrow$$




one fermi surface

## Part 5. How to realize in atomic gases

A. Hetero-nuclear mixture of two species

$$^{6}\mathrm{Li} + {}^{40}\mathrm{K}\,,\ {}^{6}\mathrm{Li} + {}^{86}\mathrm{Rb}\,,\ \ldots$$

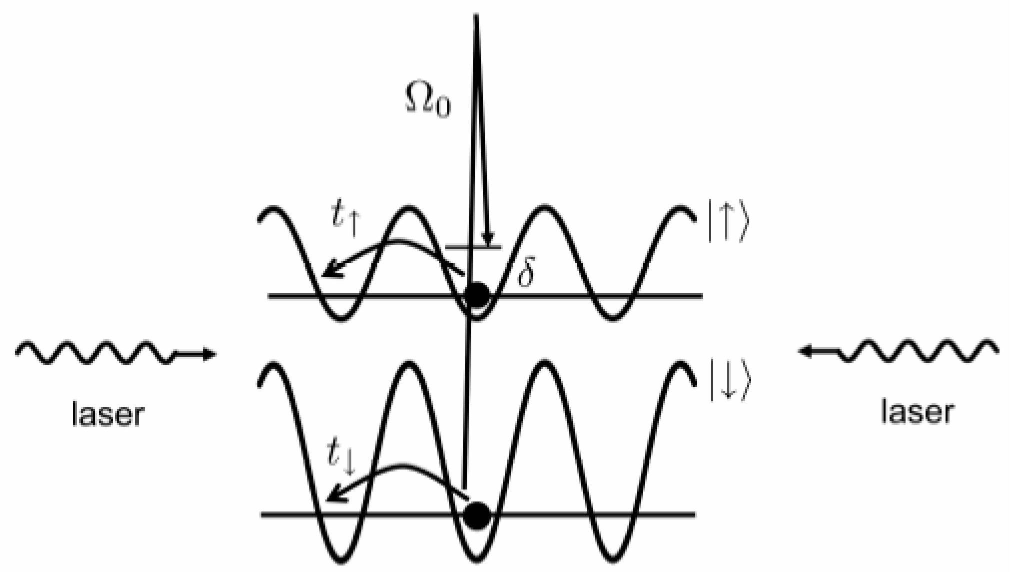
Make two species of unequal densities!

Hetero-nuclear resonance to generate attractive interactions.

B. Lattice atomic gases

# Proposed experiment of fermionic atoms on lattice

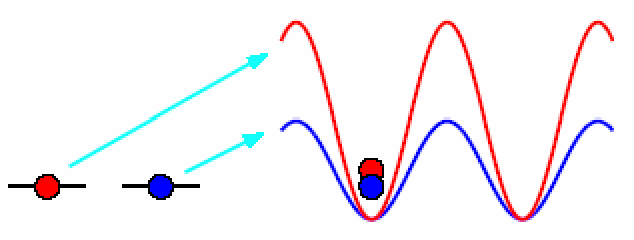
[WVL, F. Wilczek, and P. Zoller, PRA (2004)]




incoherent & different densities or coherent by Rabbi oscillation but detuned

↓

*mismatched fermi surfaces*



hopping matrix elements:

$$t_\uparrow \gg t_\downarrow\,, \quad t_\alpha \propto \frac{1}{m_\alpha}$$

# Effective range in real atomic gases

From D. Petrov, talk given at KITP Conference: Quantum gases 2004:

| | $R_e$ [Å] | $B_0$ [G] | $\Delta_B$ [G] | $\partial E_{res}/\partial B$ | $a_{bg}$ [Å] | $R^*$ [Å] |
|---|---|---|---|---|---|---|
| $^6$Li | 30 | 543.25 | 0.1 | $2\mu_B$ | 32 | 19000 |
| $^{23}$Na | 45 | 907 | 1 | $3.7\mu_B$ | 33 | 260 |
| $^{87}$Rb | 85 | 1007.4 | 0.17 | $2.5\mu_B$ | 60 | 320 |
| $^{133}$Cs | 100 | 19.8 | 0.005 | $0.55\mu_B$ | 160 | 13000 |

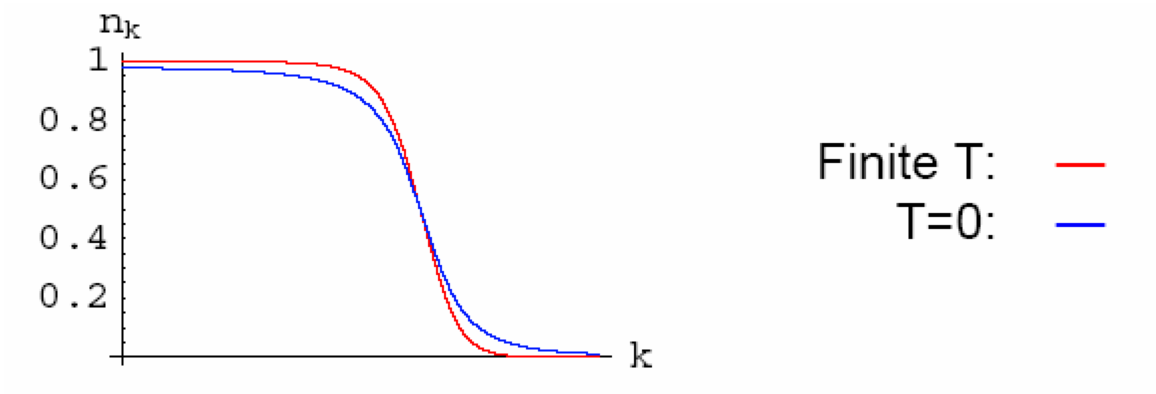
[http://online.itp.ucsb.edu/online/gases_c04/petrov/]

Gas density: $n \sim 10^{14}\text{cm}^{-3} \quad \Rightarrow \quad k_F^{-1} \sim 1.0\mu\text{m}$

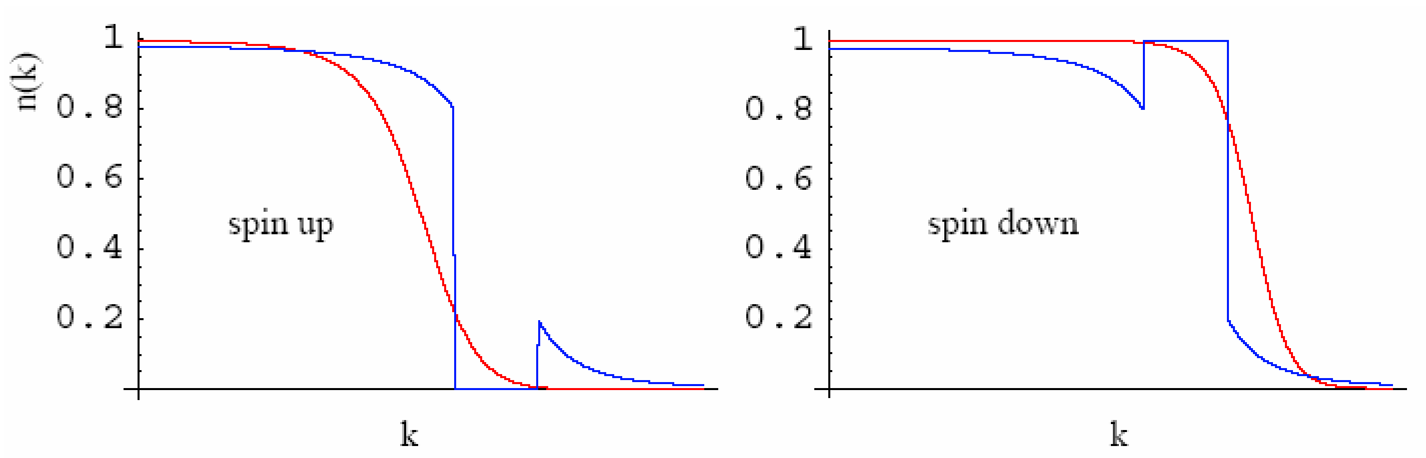
# Signature of breached pair superfluidity

(A quantum phase transition from BCS to BP)

BCS




BP



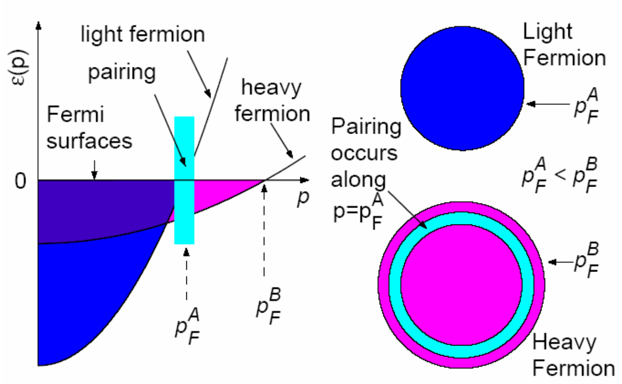
## Key features of Breached Pair

- coexisting superfluid and normal components at T=0;
- phase separated in momentum space;
- both gapped and gapless quasiparticle excitations.

## relevances to reality

- realizable with cold atoms;
- may occur as a color superconductor in quark matter such as neutron stars
- "… other scenarios for uncondensed electrons should be considered, such as 'interior gap [BP] superfluidity'" for the heavy-fermion superconductor CeCoIn5 [*quote* M. A. Tanatar, Louis Taillefer, et al. cond-mat/0503342]

# Summary




Breached Pair Superfluidity

*[courtesy of Phys. Rev. Focus (Jan 2005)]*